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Dredged material disposal in open water - the physical process and short term modeling

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ABSTRACT

Dredged material disposal in open water - the physical process and short term modeling

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With increasing awareness of the environment and of the anthropogenic contribution to the state of our waters it has become important to know the effects of dredged material disposal in open water. These operations are regulated by the Swedish Environmental Code and with time the demands of awareness of the effects of these operations have become higher.

There are many models describing suspended material spreading after disposal but not many that simulate the actual disposal and short term spreading of dredged material. This study focuses on the initial processes that govern the amount of material that go into suspension and what parameters affect the short term spreading. The model used for simulations is Short-Term Fate (STFATE).

Distribution of disposed material is divided into three phases; the convective phase which is the time from disposal to the bottom impact, the dynamic collapse which describe how the material spreads at the bottom after impact and the passive diffusion phase which is a long term process. STFATE only deals with the first two processes.

The aim is to understand the physical processes so that disposal operations in the future can be better planned and performed and the negative effects on the environment reduced. A literature review has been done in order to gather the existing knowledge on the subject. Simulations in STFATE have been performed to name the most sensitive parameters that govern initial spreading.

In simulations the water depth, ambient water velocity, disposed volume, fraction of clumps and moisture content of the material has been varied to study their sensitivity to changes. The results were plotted and interpreted regarding the distance traveled by the material from the point of disposal, the final radius of the material, the amount of suspended solids after 3600 seconds and the maximum deposition thickness.

The results show that the model is most sensitive to the amount of clay in the disposed material and of the moisture content. STFATE is a model that is quick to run and that does not require an extensive amount of input. With the right input it can be useful to get a quick overview of the situation.

Keywords: Dredging, STFATE, short term fate of disposal, sediment, convective phase, dynamic collapse

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REFERAT

Tippning av muddermaterial i öppet vatten – Fysikaliska processer och modellering av den initiala spridningen

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Med en ökad medvetenhet och kunskap om miljön och de antropogena bidragen till våra vatten har det blivit viktigt att känna till effekterna av tippning av muddermaterial i öppet vatten. Dessa operationer regleras av Miljöbalken och med tiden har kraven på kunskap om vilka effekter dessa operationer kan ha på miljön blivit större.

Det finns idag många modeller som beskriver spridningen av tippade muddermassor i öppet vatten långsiktigt, men få som simulerar själva tippningen och de initiala effekterna som styr spridningen. Det här arbetet fokuserar på de initiala processerna som styr andelen material som går i suspension samt vilka faktorer som påverkar den initiala spridningen. Modellen som använts är Short-Term Fate (STFATE).

Spridningen av tippat muddermaterial delas in i tre faser. Den konvektiva fasen beskriver tiden från tippningsögonblicket till dess att massorna kolliderar mot botten. Den dynamiska kollapsen börjar med kollisionen mot botten och slutar då materialet slutat sprida sig ut från kollisionspunkten. Sist kommer den passiva diffusionsfasen som pågår under lång tid efter tippningen. STFATE behandlar endast de två initiala faserna av spridningen.

Syftet med arbetet är att förstå de fysikaliska processerna så att framtida tippningar kan planeras och utföras med reducerade negativa effekter på miljön. En litteraturstudie har genomförts för att sammanfatta den existerande kunskapen på området. Simuleringar har sedan genomförts med STFATE för att bestämma de känsligaste parametrarna som styr spridningen.

Vattendjupet, vattenhastigheten, tippad volym, andelen klumpar och vattenhalt i det tippade materialet är parametrar som varierats i simuleringarna. Resultaten redovisas i diagram och bedöms med avseende på hur långt materialet färdats från tippningspunkten, hur stor radie molnet har efter att det lagt sig på botten, andelen sediment som är i suspension efter 3600 sekunder och den maximala tjockleken av det sedimenterade materialet.

Resultaten visar att modellen är mest känslig för andelen lera i materialet samt materialets vattenhalt. STFATE är en modell som är snabb att använda och som inte kräver omfattande indata. Modellen kan med rätt indata vara användbar då man vill få en snabb överblick av situationen.

Nyckelord: Muddring, STFATE, tippning av sediment, konvektiv fas, dynamisk kollaps

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PREFACE

This master thesis is performed as part of the Aquatic and Environmental Engineering program at Uppsala University and comprises 30 ECTS points. The project was carried through in cooperation with Envipro Miljöteknik in Stockholm and Danish Hydraulic Institute (DHI) in Lund.

Tomas Hjort (Envipro) and Olof Liungman (DHI) were my supervisors. Christoph Hieronymus at the Geophysics department at Uppsala University was my subject reviewer.

I first contacted Envipro in April 2007. They came up with the original idea to this project in their work with harbor construction projects where the demands in choosing a suitable location for the dredged material to be disposed have augmented.
Since the character of the project is rather technical DHI, who have previous experience in suspended material spreading, were also asked to supervise.

First I would like to thank Tomas, Olof and Christoph for their valuable comments, ideas and input to my work. I send a special thanks to Tommy Giertz at DHI who helped me get in touch with Olof at a time when I was the most desperate. I thank Sture Lindahl at the Swedish Meteorological and Hydrological Institute (SMHI) who helped me with literature and Mikael Törnqvist at Boskalis Sweden AB who gladly answered all my stupid questions regarding barges.

My colleagues at Envipro Miljöteknik and Hifab in Stockholm have a special place in my heart for letting me join in meetings, accompany them on field trips and making me feel like one of them. Last but not least I would like to send my thanks to my family, friends and the Geocentrum study room crew who would listen to my complaints, comfort me and always make me laugh.

Ebba Svahnström

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POPULÄRVETENSKAPLIG SAMMANFATTNING

Tippning av muddermaterial i öppet vatten – Fysikaliska processer och modellering av den initiala spridningen

Ebba Svahnström

Muddring utförs i hamnar och farleder för ökad framkomlighet och underhåll. Muddringsarbeten utförs genom att sediment avlägsnas till exempel med en skopa och läggs på en pråm. Är sedimenten fria från föroreningar kan det bli aktuellt att tippa de upptagna massorna ute till havs. Massorna transporteras då ut till en utvald tipplats där de släpps ut.

Dessa operationer regleras i Sverige av Miljöbalken. Då kraven ökar på val av tipplats och på medvetenhet om spridningen av muddermassorna efter tippningen blir det viktigt att känna till effekterna av tippning av muddermaterial i öppet vatten.

Det finns idag många modeller som beskriver den långsiktiga spridningen av tippade muddermassor i öppet vatten, men få som simulerar själva tippningen och tittar på de kortsiktiga effekterna. Det här arbetet fokuserar på de initiala processerna som styr hur stor del av materialet som går i suspension samt vilka parametrar som påverkar den initiala spridningen. Modellen som använts är Short-Term Fate (STFATE). STFATE är utvecklad inom programmet för forskning om muddermaterial (Dredging Material Research Program) av US Army Corps of Engineers. Programmet bygger på modeller utvecklade under 70-talet.

Spridningen av tippat muddermaterial delas in i tre faser. Den konvektiva fasen beskriver tiden från tippningsögonblicket till dess att massorna kolliderar mot botten. Då materialet har så pass mycket högre densitet än det omgivande vattnet sjunker det mot botten samtidigt som det transporteras horisontellt med omgivande vattenströmmar. Olika faktorer bidrar till spridningen av materialet. Några exempel på dessa är vattendjupet, vattenhastigheten och sedimentkompositionen.

Den dynamiska kollapsen börjar med kollisionen mot botten och slutar då materialet slutat sprida sig ut från kollisionspunkten. Efter kollisionen sprids materialet radiellt ut från nedslagspunkten. Den passiva diffusionsfasen pågår under lång tid efter tippningen och tar vid efter att materialet lagt sig till rätta på botten. STFATE behandlar endast de två initiala faserna av spridningen.

Syftet med arbetet är att förstå de fysikaliska processerna så att framtida tippningar kan planeras och utföras med reducerade negativa effekter på miljön. En litteraturstudie har genomförts för att sammanfatta den existerande kunskapen på området.

Litteraturstudien visar att sedimentegenskaperna hos det orörda materialet har betydelse, men också att egenskaperna ändras vid muddringsarbeten samt under transport och tippning. Det gör att det tippade materialets egenskaper kan vara helt annorlunda än de var innan muddringen. Den valda tippningsplatsens egenskaper och tidpunkt för tippningen har också betydelse. Tippningar i Sverige utförs ofta på hösten då ekosystemet är som minst känsligt för förändringar. Andra krav som ofta ställs är att platsen där man planerar att lägga muddermassorna är en ackumulationsbotten som ligger på mer än 40 meters djup. Det tippade materialet får ej heller placeras i områden som kan antas ha skyddsvärda naturvärden.

Simuleringar har genomförts med STFATE för att bestämma de känsligaste faktorerna som styr spridningen. Vattendjupet, vattenhastigheten, tippad volym, andelen klumpar och vattenhalt i det tippade materialet är parametrar som varierats i simuleringarna. Resultaten redovisas i diagram och har bedömts med avseende på hur långt materialet färdats från tippningspunkten, hur stor radien på molnet är när det lagt sig på botten, andelen sediment som är i suspension efter 3600 sekunder och den maximala tjockleken av det sedimenterade materialet.

Resultaten visar att modellen är mest känslig för andelen lera i materialet samt materialets vattenhalt. Ökande hastighet på vattenströmmar ledde till att materialet transporterades längre bort från utsläppsplatsen. En större tippad volym gav en större radie och ett tjockare täcke på botten.

STFATE är en modell som är snabb att använda och som inte kräver omfattande indata. Modellen kan med rätt indata vara användbar då man vill få en snabb överblick av situationen.

1 INTRODUCTION

When building or performing maintenance in water, such as in harbours and channels, it is often necessary to dredge. Dredging means removing bottom sediments, for example in order to increase the depth. After the dredging operation is finished there is dredged material that needs to be taken care of. One way, if the sediments are free from contaminants, is to place the material on the bottom of the sea at a carefully selected position. The sediments are transported to the chosen location by a barge and thereafter disposed into the sea.

Finding a place suitable for disposal of dredged material is difficult. There are numerous factors to account for. Both properties of the sediment, of the disposal location and of the dredging method are important. The huge amount of parameter values needed make it difficult building a universal model.

Figure 1 Mechanical dredging and placement of excavated sediments on a barge at Värmdö Garpen. (www.sjofartsverket.se/pages/9266/allmant3.gif)

A number of models have been created to describe the spreading process of suspended material but not much focus has been on the initial processes that govern the amount of material that will go into suspension. The rule of thumb is to assume that about five percent of the material will go into suspension on the short term (Communication with Anders Jensen at DHI, Denmark).

The aim of this study is to test the sensitivity of a dredged material disposal model by varying different factors. Part of the project consists of a literature review describing the physics of the sinking process and existing models. The model Short-Term Fate (STFATE) is then described and evaluated in a series of simulations. Focus is on describing the short term distribution of dredged material disposed from a stationary barge into open water.

2 BACKGROUND

Dredging and disposal of dredged material is considered to significantly change the environment. These kinds of actions are in Sweden regulated by the Swedish Environmental Code.

The open water disposal is often planned for the autumn since the ecosystem is considered to be too sensitive during the summer period (the 15th March to the 31st of August). Demands on the chosen disposal location are usually that it should be deeper than 40 meters and that its volume is large enough to handle the dredged material. The sea bed at the disposal location should have similar sediment composition as the dredged sediments and it should be an accumulation bottom with small bottom currents. That means that erosion from the bottom, and thereby resuspension, will be small.

The Swedish Board of Fisheries often has demands regarding safety measures in order to minimize effects on fish and fishing during and after disposal of dredged sediments. The dredged material, when suspended, can cause an increased turbidity which in turn may decrease the primary production and reduce the quality of life for the fish. Sedimentation may lead to a smaller availability of playing and nursing places.

Most of the research on the short term fate of dredged material being disposed in open water have been done and gathered in the Dredged Material Research Program (DMRP) performed by the U.S. Army Corps of Engineers between the years of 1973 and 1978. DMRP provides methods and guidelines to minimize the negative effects that disposal operations might lead to.

2.1 DESCRIPTION OF THE SINKING PROCESS

After disposal of the dredged material in open water, the sinking process is commonly described by three phases. The first phase is called the convective phase and describes the dredged material from the moment of disposal to the moment of bottom contact. After impact on the bottom the material collapses in a dynamic collapse. The third phase is the passive diffusion phase which takes place after the dredged material has settled. Figure 2 shows a model of the initial sediment transport.

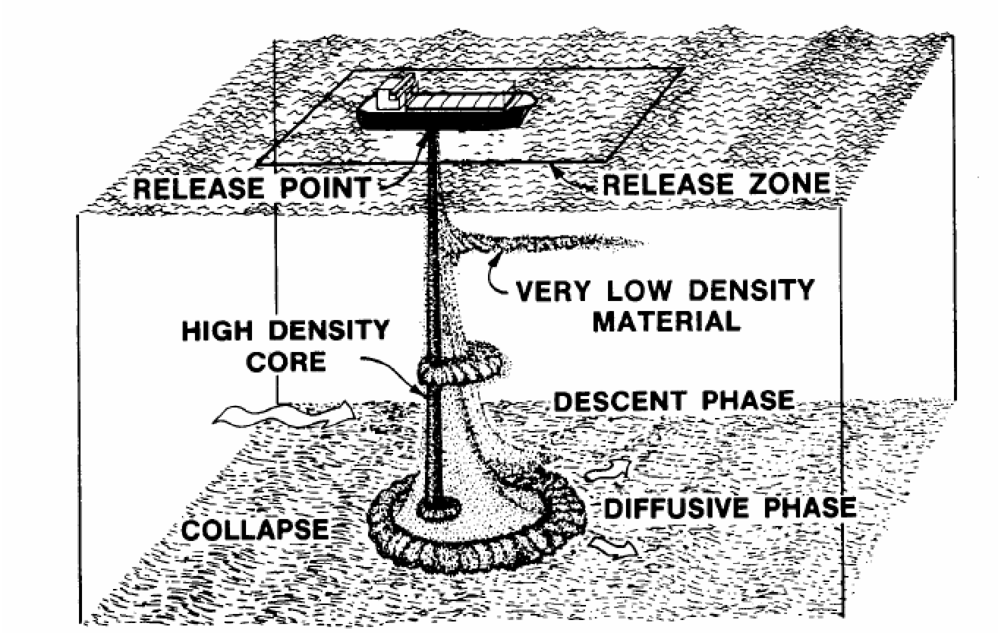

Figure 2 Dredged material being discharged from a stationary barge. (Pequegnat et al. 1981)

When the material is disposed it starts sinking towards the bottom. If the moisture content of the disposed sediments is high and the density is much higher than the density of ambient water, the dredged material will sink as a density current. The current will contain a range of particle sizes, from really fine clay particles up to big clumps or aggregates (Raymond 1986). On its way down to the sea bed, the current grows from entraining ambient water. Some of the material is lost to the surrounding water due to turbulent shear forces, and carried away by water currents. These processes are described in more details in section 3.1. (Brandsma, 1976)

When the disposed material reaches the bottom or a layer in the water column with the same density as the sinking mass, it collapses. The material which does not stop at the collision point continues spreading radially in a density- and momentum driven surge until so much of its energy is lost that the particles settle. The dominating long term process form this point on is passive diffusion which is not modelled or studied further in this report. (Raymond 1986)

2.2 THE HISTORY OF MODEL DEVELOPMENT

Before building the first model one usually relied on data and observations from other disposal operations to estimate the short term spreading of dredged material. Because of the great amount of parameters that could differ between disposal sites this was an uncertain method. A model flexible enough to simulate local conditions, dredging and disposal techniques and sediment properties that was still easy to use was needed.

Koh and Chang (1973) developed a model for short term spreading of dredged material disposal in open sea that seemed promising, but that could not model dynamic environments such as estuaries (Holliday and Johnson, 1978).

Brandsma and Divoky used the model developed by Koh and Chang and added a modification for long term diffusion from a model by Fischer 1972. From this work came two new models (Brandsma 1976), one called Disposal from an Instantaneous Dump (DIFID) and one called Disposal From a Continuous Discharge (DIFCD). A third model which is a combination of these two was then developed for simulation of dredged material Disposal from a Hopper Dredge (DIFHD). Material disposed by a hopper usually has high moisture content and the disposal operation is said to be semi-continuous (Johnson 1990). The latest update of these models is the Short-Term FATE model (STFATE) which models instantaneous disposal and disposal from hopper dredges and is a part of the ADDAMS system (Automated Dredging and Disposal Alternatives Management System).

Another ADDAMS model is the Long-Term FATE model (LTFATE) which models the long term spreading after a open water disposal. Usually the disposal operation consists of numerous discrete disposals with the barge over a long time. For this reason the MDFATE (Multi-Disposal FATE model) was created. The MDFATE uses simulations from both previous mentioned models (STFATE and LTFATE) to simulate the effect of numerous disposals over a certain period. (Schroeder et al., 2004)

There are also other models that describe the disposal of material into open water. These models are used for material disposal of sediments with high moisture content dredged with hydraulic methods. The disposal of this kind of material, where the source is assumed to be continuous, is often simulated as a negative buoyant jet (that sinks due to its density being greater than the surrounding fluid). In this way the model can simulate a release from a barge where the material sinks towards the bottom while spreading horizontally. An example of this kind of model is Cornell Mixing Zone Expert System (CORMIX) which describes the disposed material as the mirror image of a buoyant plume from a bottom discharge. This is a fundamental assumption for the Dredging Operations Mixing Zone Model (DROPMIX) which is a development of CORMIX. (Chase, 1994)

3 SHORT-TERM FATE MODEL

The STFATE model is used to determine the short term spreading of dredged material disposed in open water and its immediate effect on water quality. Short term spreading is defined as spreading controlled by material properties without being influenced of ambient currents. Long term spreading is defined as the spreading controlled by passive diffusion depending on ambient water properties. STFATE mathematically models the fate of the dredged material, within the first hours after disposal. The model is capable of estimating the amount of solids that go into suspension, the concentration of a dissolved contaminant and the initial deposition thickness on the bottom. The output from STFATE might be used as input to long term models like LTFATE. Both STFATE and LTFATE are used in MDFATE to model numerous disposals at the same site over a longer period. (Schroeder et al. 2004)

3.1 TECHNICAL DESCRIPTION OF THE PROCESSES IN STFATE

As described in section 2.1 the sinking of the disposed material is described as three separate phases. The disposal is assumed to be instantaneous and the sinking mass is described as clouds of hemispheric shape that maintain their identity by the formation of a vortex ring structure (Figure 3).

If the shape of the disposed material initially differs from a hemisphere it will soon take that shape since the vortex ring has a tendency to entrain fluid from behind it. (Brandsma, 1976)

The method of modelling the disposed masses as separate clouds was developed by Johnson and Fong (1995) when performing validation studies for an earlier version of STFATE. After the impact of the disposed material at the bottom, the masses are assumed to spread radially from the point of impact with decreasing energy with distance. When performing large-scale laboratory tests it could be shown that the disposed material would leave the vessel in globs and add additional energy to the spreading bottom surge. The idea then came up that the sinking material should be modelled as a sequence of convecting hemispheric clouds with different sinking velocities and densities. This can be used for example when disposing material from a Hopper with several bins assuming that the material in one bin represents one cloud (Hales, 1996). In STFATE there is a choice of how many bins that opens simultaneously. This way of modelling the material makes it possible to more correctly account for consolidation during transport. (Palermo, 1998)

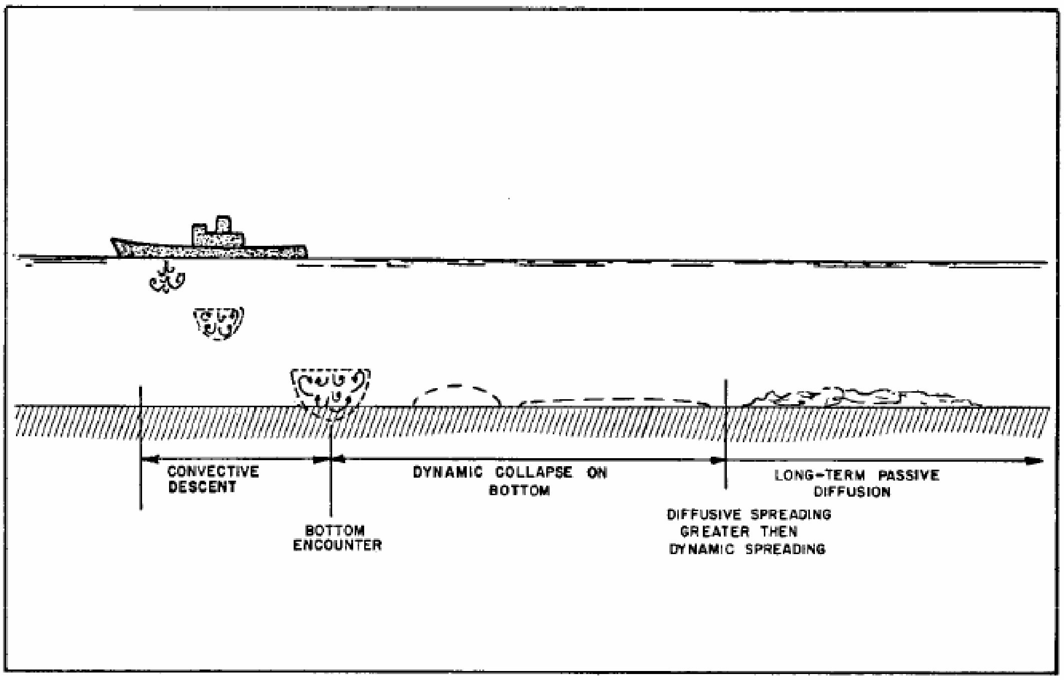

Figure 3 The three phases of dredged material disposal. The disposed material is modelled as sinking hemispheres with the curved arrows showing the effect of the vortex ring (Brandsma, 1976).

3.1.1 The Convective Phase

The equations governing the convective phase include those for conservation of mass, momentum, buoyancy, solid particles and vorticity. Equations and more detailed information are given in appendix 2.

Due to its high density and the initial momentum (from the disposal) the hemispheres will sink towards the bottom. Ambient water will be entrained in the plume and make the cloud volume grow. Some of the really fine particles will be stripped from the sinking plume to ambient water making the particle concentration of the cloud decrease (equation 1).

$$Time\ rate\ of\ change\ of\ mass = Entrainment - Stripped\ particles \qquad (1)$$

In the equation of conservation of mass the volume of the particle cloud is defined as the volume of a hemisphere. The entrainment of water depends on the velocity difference between the cloud and the ambient fluid (equation 2). The radius of the hemisphere is denoted a. The entrainment coefficient (α) depends on properties of the sediment cloud, properties of the ambient water and turbulence inside and outside of the cloud. $\vec{U}$ is the velocity of the cloud and $\vec{U}_a$ is the speed of the ambient water.

$$E = 2\pi a^2 \alpha \left| \vec{U} - \vec{U}_a \right| \qquad (2)$$

The rate of solids passing out of the cloud is a product of on the concentration of particles in the cloud, how fast they fall and the vertically projected area of the cloud (Equation 4, Appendix 2). If the concentration is high, the projected area is big and/or the fall velocity of the solid particles is high more particles will leave the cloud. The fall velocity is a constant depending on the kind of particles disposed (clumps sand, silt or clay). A small settling coefficient (Equation 4, Appendix 2) will also give a higher amount of stripped particles. The settling coefficient describes the materials inclination to settling and is set to a different value for each particle fraction. Figure 4 shows the coordinate system of a sinking dredged material cloud, the velocity profile and density profile of the ambient water. (Brandsma, 1976)

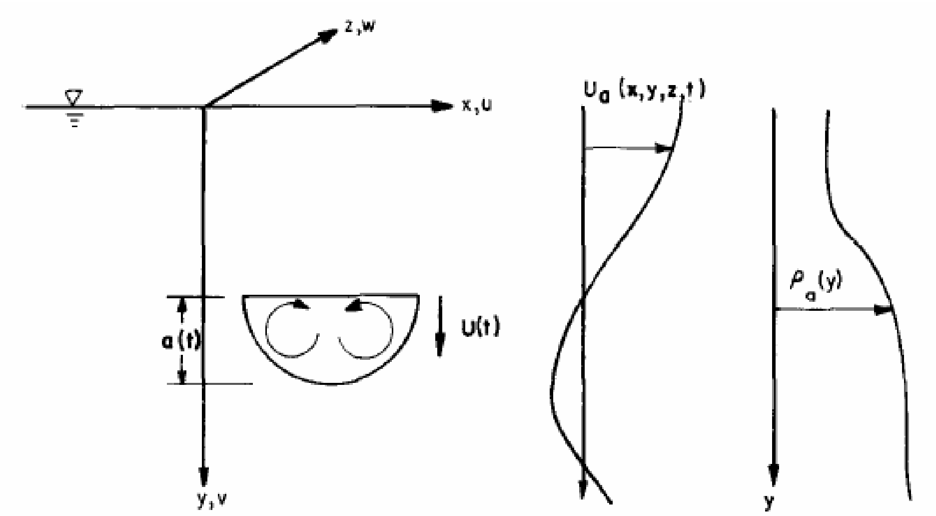

Figure 4 A sinking cloud in a density stratified water where the ambient water velocity (U_a) varies with depth. (Brandsma, 1976)

The second governing equation describes the time rate of change of momentum. It is a function of the buoyancy, the drag, the rate of ambient fluid entrainment and the rate of solids leaving the cloud. The buoyancy is defined in equation 3. Since the material is much denser than the ambient fluid we are dealing with negative buoyancy which increases the time rate of change of momentum (think of a rising bubble of air in water and turn the picture around 180 degrees). Drag is the force that resists the movement of a solid object through a fluid. The entrainment also adds to the momentum, depending on the direction of ambient water flow, while solids leaving the cloud decrease it (equation 4). (Brandsma, 1976)

$$Buoyancy = \frac{2}{3}\pi a^3 g(\rho - \rho_a) \qquad (3)$$

$$\begin{aligned}&Time\ rate\ of\ change\ of\ momentum = Buoyancy - Drag + \\ &Momentum\ addition\ by\ entrainment - Momentum\ removal\ by\ stripped\ particles\end{aligned} \qquad (4)$$

The time rate of change of the solid volume of material component *i* (P_i) depends on the particle flow out of the cloud. P_i is defined as the volume of the cloud times the volume fraction of solid particles (C_{si}) (equation 5).

$$P_i = \frac{2}{3}\pi a^3 C_{si} \qquad (5)$$

The last governing equation is that for vorticity. The total vorticity is the mechanism that helps the cloud to keep its identity. It is also important when determining the amount of entrainment of ambient water.

The total vorticity is created by shear forces at the boundaries. When the cloud is in the ambient water there are two possibilities, except for bottom contact. In a non stratified environment, with a homogenous density, the cloud keeps its total vorticity although the vortex strength is decreasing with cloud growth. In a stratified fluid the density gradient will act to decrease the total vorticity.

During the convective descent phase fine material may leave the cloud due to ambient water currents or to movement of the barge and stay in the upper parts of the water column.
The material that leaves the sinking plume is considered to spread with Gaussian particle distribution and move by ambient water currents (equation 22, appendix 2). Experiments have shown that fine particles may also leave the density current at the impact at the bottom. These particles are also considered to have a Gaussian particle distribution (Johnson, 1990 and Johnson, 1993).

The output from the calculations of the convective phase is used as input for calculations on the dynamic collapse. Given a set initial conditions the equations can be solved with several numerical methods.

3.1.2 Dynamic Collapse at the Bottom

The dynamic collapse is not the focus of this project. Nevertheless it is an interesting and important part of the disposal operation and planning and therefore it will be discussed below.

Depending on the ambient water stratification the cloud will collapse either at the stratification boundary or continue and collapse at the sea bed (Brandsma, 1976). In Sweden dredging operations are in most cases performed during autumn when the water is well mixed. For that reason this essay will only discuss the dynamic collapse at the bottom. Further information on collapse at a stratification boundary can be found in the work of Brandsma (1976).

As the sediment cloud goes through the convective phase it gains mass. Simultaneously the particle concentration is decreasing. Its horizontal velocity approaches the velocity of the ambient water and the vorticity approaches zero.

When the cloud collapses at the bottom, its cross section gets elongated in the horizontal direction and is now described as a half ellipsoid instead of a hemisphere (figure 5a). After impact the dredged material continues to spread radially until it has lost so much energy that the particles start to settle. The collapse phase terminates when the rate of spreading becomes less than an estimated rate of spreading due to turbulent diffusion. (Johnson, 1990)

The sediment cloud is assumed to remain symmetrical and the bottom is assumed to be horizontal. The equations describing collapse at the bottom are practically identical to the ones describing collapse at a stratification boundary except for some changes of geometry and to account for the reaction and friction forces at the bottom. The governing equations are described in appendix 2. Conservation of mass is defined as before but the volume is now the volume of half of an ellipsoid. The equation of momentum is identical to the momentum equation for the convective decent except for an addition of a reaction force at the bottom (that acts to decrease it). (Brandsma, 1976)

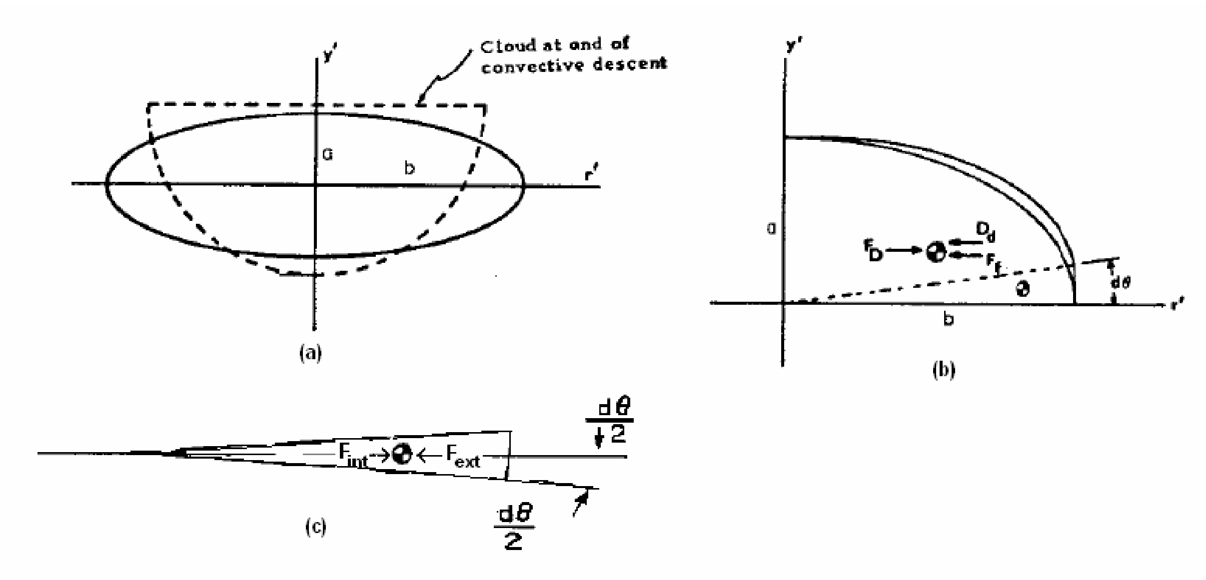

Figure 5 A collapsing sediment cloud. Figure 5a shows how the shape of the cloud changes from a hemisphere to half a spheroid at the bottom contact (the r' axis represents the bottom). 5b shows the forces acting on the centroid of the spheroid and the centroid of a slice. 5c describes a slice of the collapsing cloud and the forces acting on the centroid of it. For symbols see text. (Brandsma, 1976)

The entrainment in this phase is due to the collapse alone since the velocity of ambient fluid at the bottom is assumed to be small. The entrainment is a product of the surface area of the cloud that is exposed to the ambient water, an entrainment coefficient and of the velocity of the tip of the collapsing cloud (equation 6).

$$Entrainment = Surface\,area \times Entrainment\,coefficient \times Tip\,velocity \qquad (6)$$

It is density differences between the inside and the outside of the cloud that drive the collapse. Because of turbulent mixing the density gradient inside the cloud is less than on the outside.

Figure 5b shows the centroid of the collapsing cloud. The centroid of a slice of the collapsing cloud with an angular dimension $d\theta$ is moving with respect to the centroid of the cloud. F_D is the internal forces minus the external forces. D_d is the form drag which is a friction force caused by the shape (form) of the cloud and F_f is the skin friction drag (a friction force depending on the properties of the surface of the cloud). In figure 5c the forces acting on the centroid of a slice of the collapsing cloud are shown. If the thin slice is considered as a free body the external force F_{ext} can be calculated by integrating the pressures over the rounded external surface of the slice. F_{ext} is the external radial force acting at the slice centroid. (Brandsma, 1976)

The pressures are assumed to be hydrostatic and the pressure at the cloud boundary is equal to the pressure of the ambient fluid, $p_a(y)$.

The pressure on the inside of the cloud is integrated on both sides of the slice and $d\theta$ is assumed to be small. The internal force (F_{int}) driving the collapse is a function of the radial position inside the cloud and is directed radially outwards. After subtracting F_{int}-F_{ext} the radial force driving the collapse (F_D) is obtained (equation 36 appendix 2).

The equation describing inertia of the slice is formulated by assuming that the horizontal velocities of the elements inside the slice are related to the radial distance (r) from the centroid of the cloud. It is also assumed that the velocity of horizontal deformation of the cloud is characterized by the velocity of the centroid of the slice. The tip velocity of the cloud is linearly related to the velocity of the centroid (dc/dt). It is the sum of the velocity due to the collapse and the velocity due to entrainment of ambient water. (Brandsma, 1976)

Some extra equations are needed to describe the momentum, buoyancy force, drag forces and bottom friction forces in the three directions (Appendix 2).

3.2 LIMITATIONS TO STFATE

In the case of disposing dredged material with high moisture content, the material is often disposed through opening a few bottom doors of the barge at the time. The whole disposal operation might then take up to a few minutes. At the same time fractions of the material might be in the form of clumps that fall quickly to the bottom. This makes the disposal time long simultaneously as the falling time for some material fractions is very short. A limitation in the model is that the total time it takes for the material to leave the barge can not be greater than the time it takes for the material to reach the bottom.

STFATE includes three sub models (for instantaneous, continuous and semi continuous disposal). They all require data on volume and fall velocity of solid fractions. Often the dredged material form big aggregates with low moisture content, especially if the sediments are mechanically dredged or when material has time to consolidate during transport. The difficulty in measuring how much of the material that is consolidated might lead to complications in modeling.

The sinking material is assumed to fall according to Stoke's law which is based on properties of the particle. If a solid fraction is assumed to be cohesive, the sinking velocity is calculated as a function of the particle concentration of that fraction. When they are not cohesive they are assumed to sink with a constant velocity. This assumption might not be realistic in the presence of turbulent forces induced by for example waves and currents.

When defining a grid for calculations there is a possibility of adding different water depths for each grid point. When simulating the dynamic collapse STFATE does not take this into consideration but simulates the bottom as being horizontal. The bottom is only allowed to slope in one direction in each grid. This means that simulations of disposal on for example a mound are not possible. However, a modification has been done for the possibility of disposing the dredged material in a depression. The depression is modeled as a rectangular hole. (Johnson, 1990)

Sometimes there is a wish to enter different grid element sizes at different locations. For example it might be interesting to have more data on the area close to the point of disposal than out at the site boundary. STFATE generates a grid system where all grids has the same size.

STFATE includes predefined coefficients describing for example the stripping of solid material from the main plume. The value of the stripping coefficient is selected so that approximately 2-5 percent of the total volume of fine material is stripped away at disposal sites of 30 meters or less. Based upon field data collected by Bokuniewicz et al. (1978), this will result in the amount of stripped material being on the conservative side. This is a limitation if one wish to examine the amount of material stripped from the plume.

One of the most important limitations of STFATE is that particles that have settled are assumed to stay there. Since no resuspension is assumed this model is only recommended for short term modeling (Johnson, 1990). This can be avoided by using STFATE in combination with a long term model such as LTFATE.

3.3 FACTORS IMPORTANT FOR MODELING

Examples of factors that might influence the spreading of disposed material are water depth, ambient water velocities and properties of the dredged material. Depending on the original sediment composition at the dredging site and of the dredging technique, the dredged material gets different properties (for example water content and shear strength). The disposal operation technique and the properties of the disposal location are also important for the description of the sinking process. A model for dredged material disposal in open water has to be able to account for local environmental conditions, sediment properties and different disposal techniques.

3.3.1 Dredging and Disposal Techniques

Two ways of dredging are mechanical (clamshell) or hydraulic dredging (suction). Mechanical dredging is most suitable for sediments with low moisture content and high shear strength. The sediments are removed in big clumps with close to in situ cohesive properties and density.

The dredged sediments are placed on a barge or a scow and transported to the location of disposal. Even though several disposals usually take place at the same location, they are considered as several discrete events. The disposal is assumed to be instantaneous after the doors have opened, and the material usually maintains its shape and reaches the sea bed relatively unaffected. Only a small amount of particles leave the density current and goes into suspension. (Palermo, 1998)

In some cases, when dealing with sediments with high moisture content and low shear strength, the sediments are removed by hydraulic dredging. The sediments are mixed with water and are then sucked up with a pump and placed on the hopper. This technique gives the sediments very high water content. The hopper transports the material to the disposal site. (Palermo, 1998)

3.3.2 Sediment Properties

To get good results when modelling sediment plumes created by dredging and disposal operations it is very important with knowledge about the geotechnical and chemical properties of the sediments. When dredging a large area the sediment composition might vary a lot from site to site within the area. It would be very costly to take that many sediment samples and therefore estimations of sediment properties are often made. These estimations may lead to significant errors in model results.

As the sediments are removed during dredging, they are mixed with water and sediment from nearby sites. This gives the dredged sediment new properties. Figure 6 shows how different actions may affect the material. (Lee, 2001)

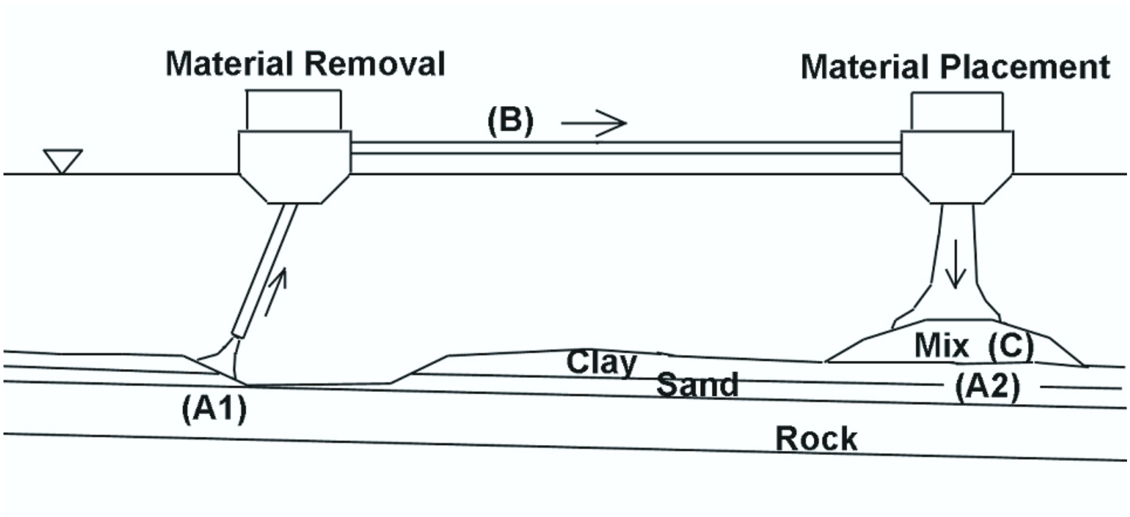

Figure 6 Different actions during dredging and disposal that may affect the sediment properties. (Lee 2001)

The original sediment (A1) is removed by mechanical or hydraulic methods. Depending on the method, the sediments will mix to different extents to give material B. The dredged material is then placed on a barge or hopper and transported to the disposal site. During the transport the material may consolidate. When the material is disposed it has changed again to form material C. Even though one aim is to dispose sediments from a dredging site at a location with sediments of similar composition, much may have changed during the operations. To give the sediment different properties at different steps when building a model is complicated since there is often a lack of data. (Lee, 2001)

Depending on the purpose of modelling, and of which model that is used, different sediment properties are important. Consistency in using a certain way of describing the material is important since there exist different units describing it. Sometimes the dredged material is given as a mass. Other time it is given as a concentration. One method is to give the void ratio of the material which is the volume of voids divided by the volume of solids. If the material is hydraulically dredged and contains a lot of water (more than its geotechnical Atterberg liquid limit) it is often described as a thick liquid with a certain concentration of particles. (Lee, 2001)

Once the material is disposed on the sea bed its bulk density has become lower due to different mixing processes during dredging and disposal. After some time the material will start consolidate on the bottom and the bulk density will go up. The maximum height of the mound created by the disposed material depends on the steepest angle at which the material can sustain itself from environmental forces and gravity. This angle is called the limiting angle of repose or shearing angle. (Johnson et al., 1999).

As mentioned in section 3.2 the fall velocity of the particles is normally calculated using Stoke’s Law. Clay is sometimes cohesive and binds to other particles. The fall velocity is then calculated as a function of the particle concentration.

3.3.3 Disposal Site Properties

Water current velocities in different water layers and at the bottom, waves and winds are parameters which can affect the spreading. Some of the parameters are seasonal, like water stratification and temperature (Holliday 1978). High salinity might lead to flocculation of particles and the formation of aggregates with higher fall velocity (McDowell, 1977).
One should also take the traffic situation into consideration since a lot of traffic might lead to turbulence and resuspension of sediments.

4 SIMULATION WITH STFATE

The aim of simulations is to test the sensitivity of the model by studying how much a change in one factor change the spreading pattern of the disposed material. Four factors are tested namely the depth, the ambient water velocity, the sediment composition and the moisture content. STFATE is accessible at the U.S. Army Engineering Website at http://el.erdc.usace.army.mil/products.cfm?Topic=model&Type=drgmat at no cost. Since it is an American product it is important to have the American settings for dots and commas on the computer used to run STFATE.

4.1 STANDARD SETUP

The program comes with 4 example files which can be run to get an idea of the possibilities of the program. One of these (Example-Barge CWA 404 Mixing Zone.DUI) was used and manipulated to set a standard scenario where different parameters could then be varied in different simulations. The program includes a number of predefined coefficients. These are used unchanged in the simulations since it has been shown that model results are quite insensitive to many of the coefficients (Johnson 1990). STFATE works in American units and table 1 shows how the units are converted.

Table 1 Conversion of units from American units to the international system of units (SI)

Multiply	By	To Obtain
cubic feet	0.02832	cubic meters
cubic yard	0.764555	cubic meters
feet	0.3048	meters
inches	2.54	centimeters
inches	25.4	millimeters
knots	1.852	kilometers per hour
miles (U.S. nautical)	1.852	kilometers
miles (U.S. statute)	1.609347	kilometers

The type of analysis chosen is “General Open Water Disposal Analysis” with “Disposal from a Split-Hull Barge or Scow”. The disposal area is defined as a square with constant water depth to make simulations as simple as possible. The area is divided into a grid with constant grid spacing (Table 2). The point of disposal must also be defined as the distance from the origin in both X- and Z-directions (figure 7). Water depth is allowed to vary in simulations, but is constant over the disposal area.

Table 2 Description of the Disposal Area.

	X-direction	Z-direction
Number of grid points	32	32
Spacing between grid points (meter)	30.48	30.48
Disposal grid dimensions (meter)	944.88	944.88
Point of disposal (meter)	365.76	457.2

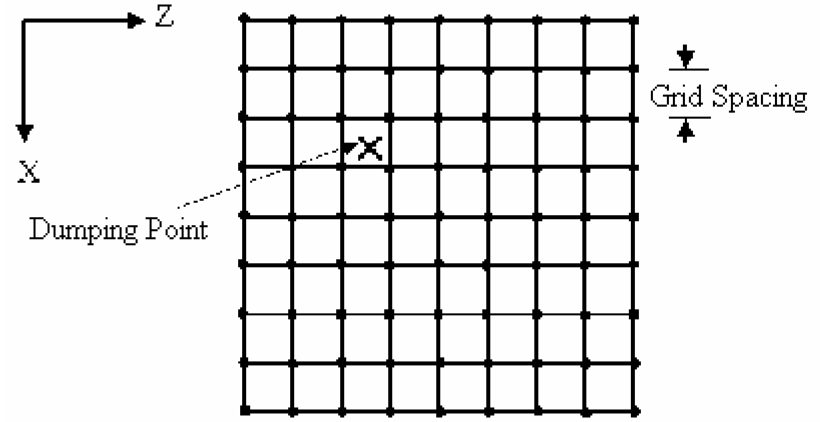

Figure 7 Grid system for the disposal area.

Water density can be entered at a maximum of 5 depths. Here two densities at different depths are used to create a density profile. Since density is not assumed to be a critical parameter the values that were already provided by STFATE were used for the different depths (at 0 meter a density of 0.9984 gram per cubic centimeter and for 40 meters and greater depths a density of 0.9993 gram per cubic centimeter). The ambient water density is thus assumed to change by less than 0.1% from the surface to a depth of 100 meters. The big density difference between the sinking sediments and the ambient water makes the small density gradient of ambient water less important in the calculations of spreading of disposed material (Pond, 1978).

The horizontal velocity of the water is varied in the simulations. The velocity is assumed to be constant throughout the entire disposal area and is varied and then plotted to test the importance of this parameter when planning and performing a disposal operation (Figure 8).

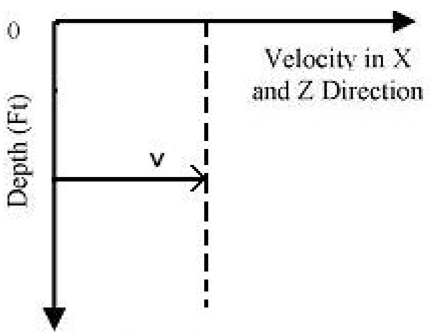

Figure 8 A constant velocity profile for a constant depth grid.

To describe the disposal operation, STFATE requires data on the length and width of the disposal bin of the barge. The sizes of barges differ a lot but the assumption is made that a Split Barge of the size 60×11.7 meters is used. The disposal bin of such a vessel is 27×7.5 meters and the drafts (pre- and post-disposal) are 3.8 and 2.2 meters. The time it takes to empty the bin is assumed to be 30 seconds. The data on the transport vessel comes from Boskalis Sweden AB and their Split Barges Frigg and Rind (Figure 9).

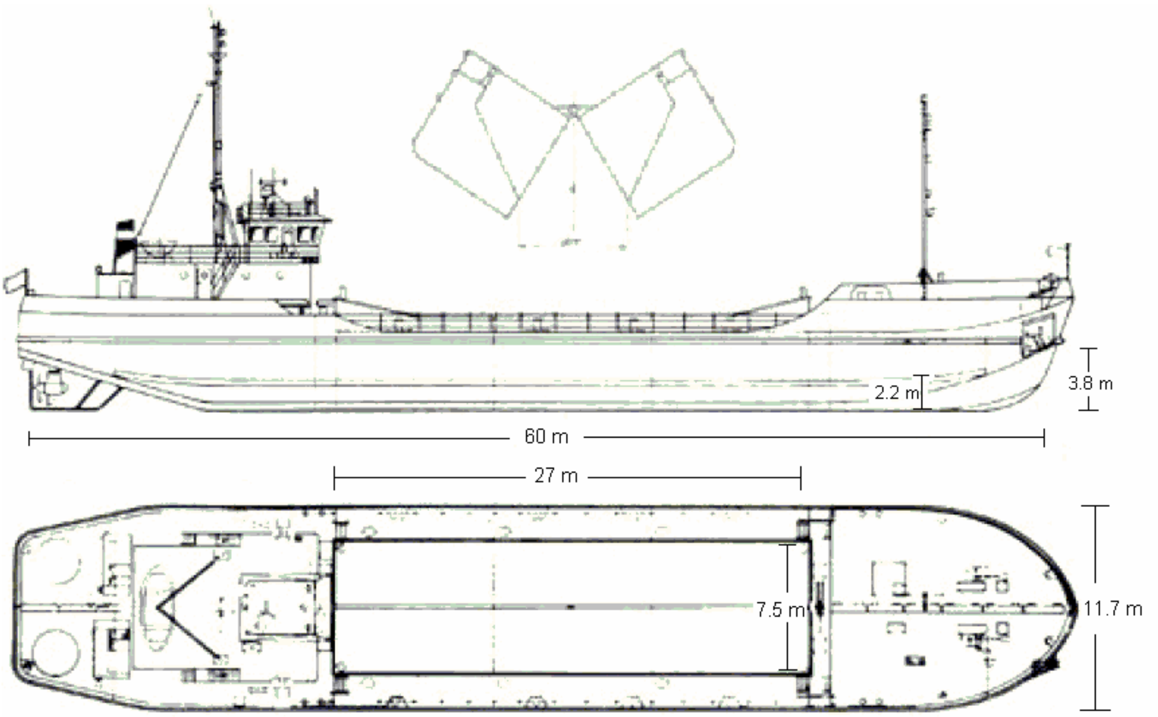

Figure 9 The figure shows Frigg/Rind, a Split Barge of the kind that is used for simulations.

Next, the dredged material data will be provided to the model. If the material is supposed to consolidate during transport to the disposal location, the number of layers of separation has to be provided to the program. The use of layers of separation is a way to take consolidation during transport into consideration. For each layer the volume and the velocity of the vessel are given. In this project the vessel is assumed to be stationary and the number of layers is set to one. The sediment composition also has to be given for each layer as volumetric fractions of clumps, gravel, coarse sand, medium sand, fine sand, silt and clay. The water density of the dredging site must also be provided.

The dredged sediments are assumed to be clay in one layer with a volume of 600 cubic meters. The ratio clumps to clay is varied. Then the effect of the moisture content on the output of the model is tested by varying the amount of clumps and clay at the expense of the moisture content.

Before the model can be run the duration of the simulation and the long term time step for diffusion has to be specified. Last the print options are set.

4.2 SIMULATION SETUP

In simulations the depth, the water velocity, the sediment composition and the moisture content are varied. The factors are varied one at the time keeping the others constant. From output files of STFATE results are collected and interpreted. Both the distance of the centroid from the barge and the final radius were determined after the completion of the collapse phase. The reason for choosing 3600 seconds after disposal when looking at the suspended solids was to account for as much suspended material as possible since material goes into suspension not only in the convective phase but also in the collapse phase. The maximum deposition thickness is measured after the cloud has settled at the bottom and is discussed even though it is an outcome of the collapse phase which is not the focus of this study.

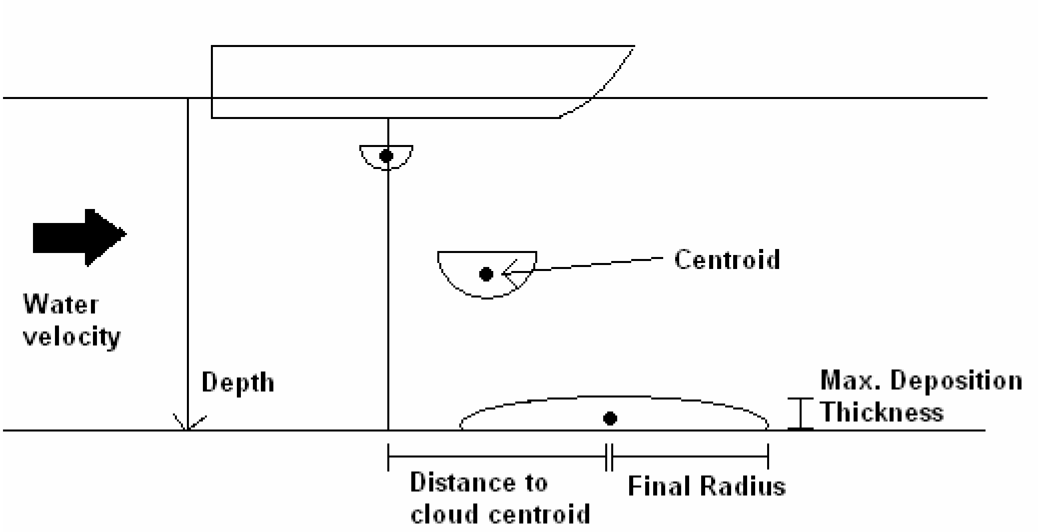

Figure 10 Definition of parameters. The distance to the centroid, the final distance, the amount of suspended material and the maximum deposition thickness are measured at the time after the collapse phase is completed.

4.2.1 Reference Values

A reference case was set up and from that case one factor at a time was varied in order to study the effects of these factors on the initial spreading. Table 3 shows the reference values in the simulations.

Table 3. Reference case values used in simulations.

Depth (m)	Velocity (m/s)	Volume (m^3)	Sediment
40	0.2	600	0.6 clumps 0.35 clay 0.05 water

4.2.2 Varying Depth at the Disposal Site

The depth was varied keeping values of water velocity and sediment composition constant. A velocity of 0.2 meters per second is a comparatively high value for velocities close to the bottom. Different values of depth that were used are 20, 40, 60 and 80 meters.

4.2.3 Varying Ambient Water Velocity at the Disposal Site

The velocities tested were 0.03, 0.2, 0.4 and 0.6 meters per second. The ambient water velocity is an interesting factor since the material after disposal might be moved horizontally due to currents and be deposited outside the planned disposal area.

4.2.4 Varying the Amount of Material Being Disposed

Looking at the effect of the volume disposed might help in future planning of this kind of operations, to see if it is better to dispose a big volume in one go or to do several disposals with smaller volumes. Nine different volumes were tested (10, 50, 200, 600, 1000, 1400, 1600, 2000 and 2500 cubic meters).

4.2.5 Varying the Sediment Composition of Disposed Material

In these simulations the ratio clumps/clay was varied (Table 4). This is interesting since the dredging technique gives the dredged material new properties. If it is mechanically dredged it will have a high clump/clay ratio. If it is hydraulically dredged it will have a small amount of clumps. First the material is assumed to be composed of only clumps. In all the simulations the removed sediments are assumed to be saturated so that the sum of all fractions always adds up to one or 100% (equation 7).

$$clay + clumps + moisture = 100\% \qquad (7)$$

Table 4 The choice of ratios of clumps to clay used in simulations.

Simulation no:	1	2	3	4	5	6	7
Clumps	0.95	0.75	0.7	0.6	0.5	0.35	0.1
Clay	0	0.2	0.25	0.35	0.45	0.6	0.85
Water	0.05	0.05	0.05	0.05	0.05	0.05	0.05

4.2.6 Varying the Moisture Content of Disposed Material

A parameter that is very interesting to vary is the water content of the disposed material. This is because the entrainment coefficient, that gives the amount of fluid that is mixed into the cloud, is sensitive to changes in the moisture content of the dredged material. It is also important since depending on the dredging method the material gets different moisture content. If the material is mechanically dredged the moisture content will be low and if it is hydraulically dredged it will be high. The simulation was done in two steps. First the amount of clay was decreased to give room for more water keeping the clump fraction constant (equation 7). Then simulations with a decreasing amount of clumps were performed keeping the clay fraction constant. In table 5 and table 6 the sediment composition in simulations is given.

Table 5 Sediment composition for the simulations with a varying amount of clay keeping the amount of clumps constant.

Simulation no:	1	2	3	4
Clumps	0.6	0.6	0.6	0.6
Clay	0.35	0.3	0.25	0.2
Water	0.05	0.1	0.15	0.2

Table 6 Sediment composition for the simulations with a varying amount of clumps keeping the amount of clay constant.

Simulation no:	1	2	3	4
Clumps	0.6	0.55	0.5	0.45
Clay	0.35	0.35	0.35	0.35
Water	0.05	0.1	0.15	0.2
Simulation no:	5	6	7	
Clumps	0.43	0.4	0.35	
Clay	0.35	0.35	0.35	
Water	0.22	0.25	0.3	

5 MODELING RESULTS

From the output files information have been extracted and plotted in Microsoft Excel. The parameters plotted are; the final distance traveled by the centroid of the cloud, the final radius of the material, the percentage of suspended solids after 3600 seconds and the deposition thickness at the bottom. The parameters are all measured after the collapse phase the scales of the plots have been chosen to make comparisons easier. The figures are ordered in the same way that they will be discussed.

5.1 DISTANCE TRAVELED BY THE CENTROID

Figure 11 shows that when increasing the water depth the distance traveled by the centroid increases linearly. The distance also seems to increase linearly with an increased water velocity (figure 12). It can be seen that changes due to increasing water velocities are bigger than those due to changes in depth.

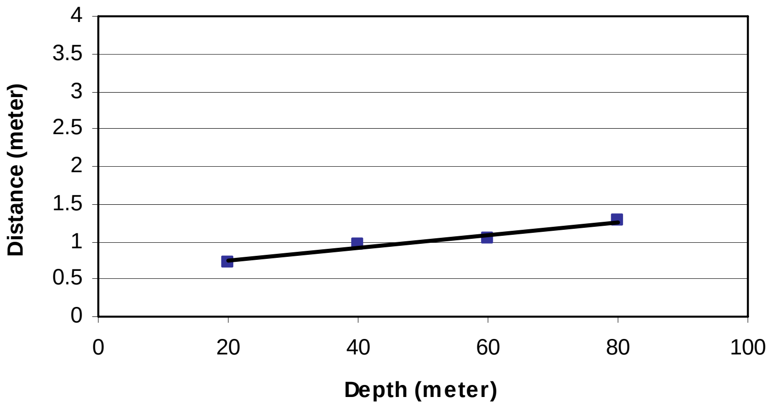

Figure 11 The final distance traveled by the cloud as a function of water depth.

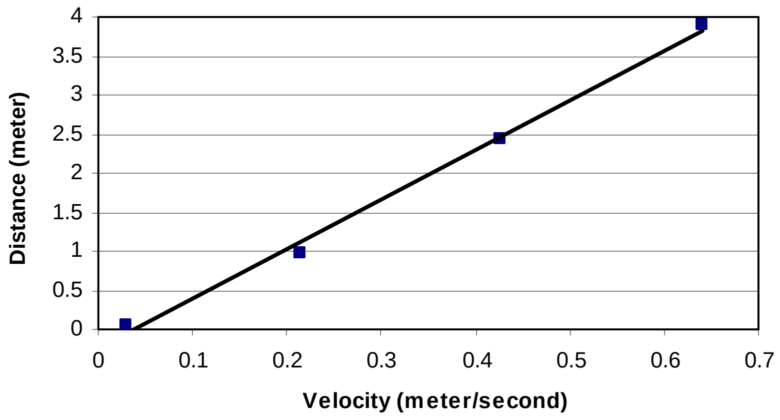

Figure 12 The final distance traveled by the cloud as a function of ambient water velocity.

In simulations with a varying volume of material disposed by the barge the sediment composition was kept constant. The simulations were done for three cases with different sediment composition. In figure 13 it can be seen that the results for the different sediment compositions are quite equal whilst the distance traveled by the centroid is much larger for a smaller amount of material being disposed.

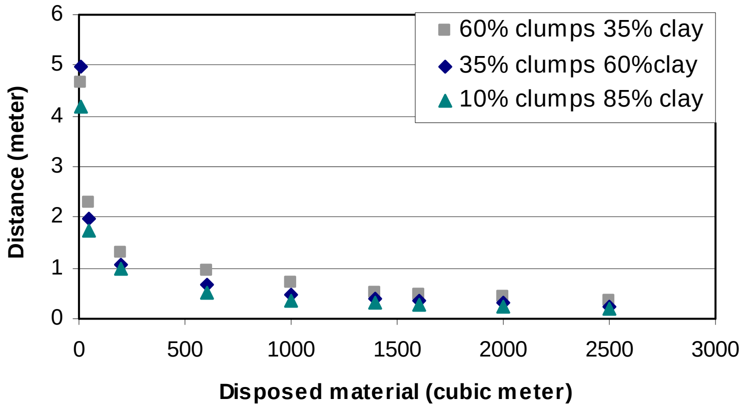

Figure 13 The final distance traveled by the cloud as a function of the volume of dredged material being disposed.

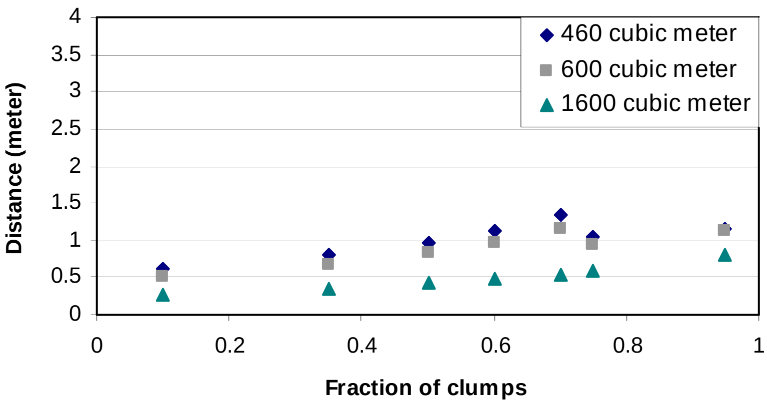

Figure 14 The final distance traveled by the cloud as a function of the fraction of clumps. The moisture content was kept constant.

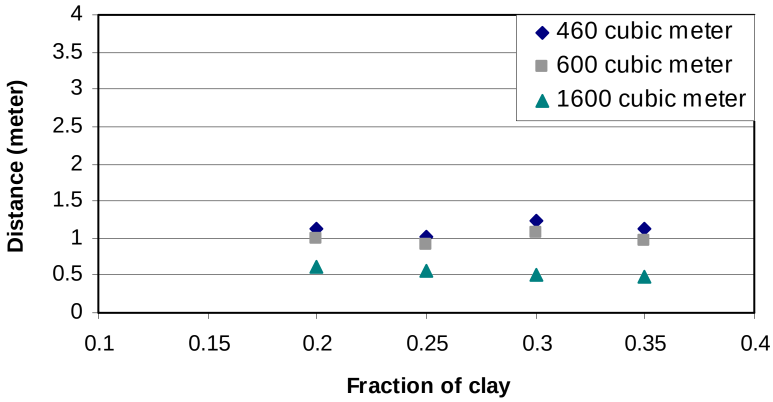

Figure 15 The final distance traveled by the cloud as a function of the fraction of clay keeping the amount of clumps constant. More clay means less moisture.

There is a general trend of decreasing distances with an increased fraction of clumps (figure 16). The simulations for 600 cubic meters of disposed material show a shift around 0.45.

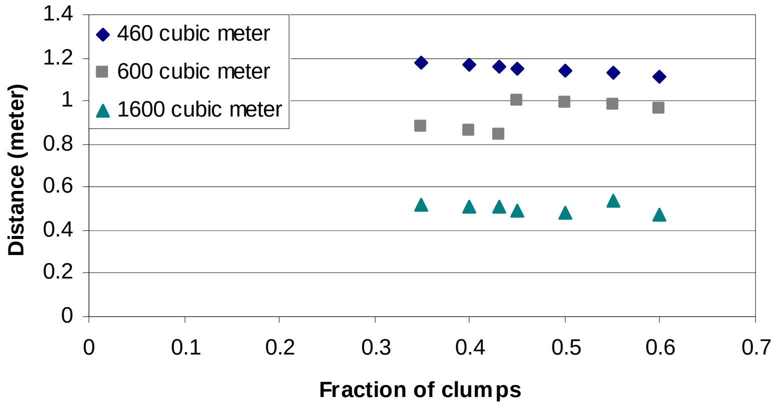

Figure 16 The final distance traveled by the cloud as a function of the fraction of clumps. The fraction of clay was kept constant.

5.2 FINAL RADIUS

The final radius is defined as the radius of the cloud just as it enters the collapse phase. In figure 17 there is once again a linear dependence with depth. The velocity does not seem to have an influence (figure 18). As for the disposed volume it increases and then seems to reach a plateau (figure 19). The radius decreases with an increased amount of clumps and increases with increasing amount of clay (figures 20 and 21). In the simulations where the clay content was kept constant and the amount of clumps was augmented there is a small increase in the final radius (figure 22).

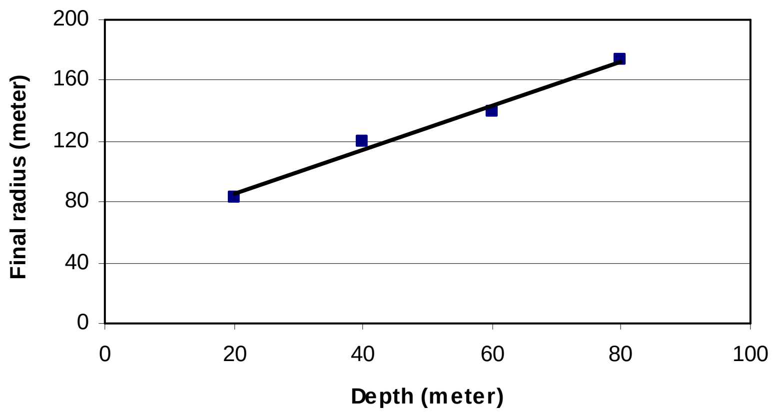

Figure 17 The final radius of the cloud at the bottom increases when the water depth increases.

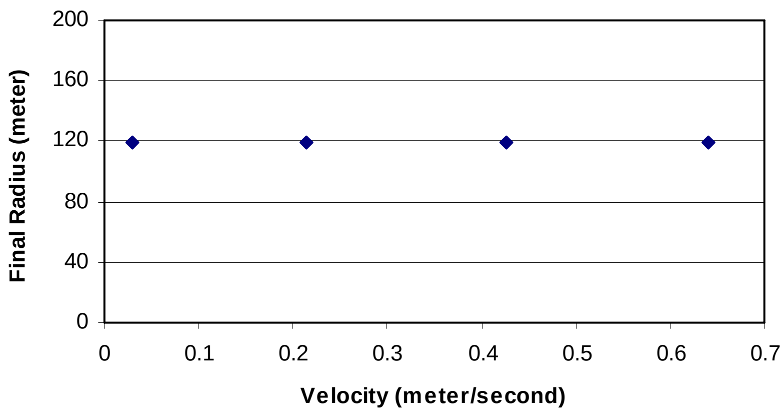

Figure 18 The final radius of the cloud at the bottom as a function of the water velocity.

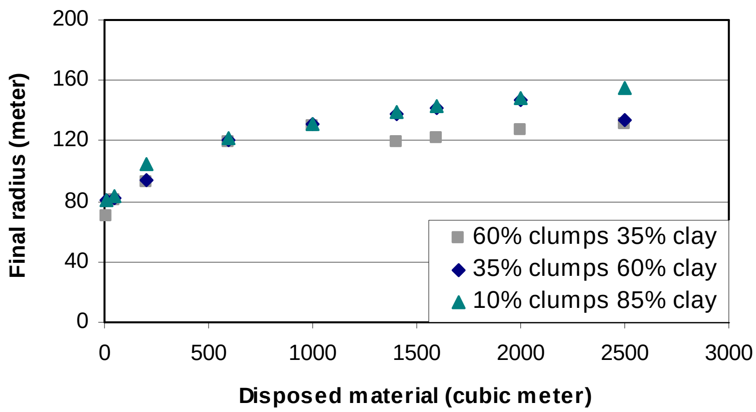

Figure 19 The final radius of the cloud as a function of the amount of material being disposed.

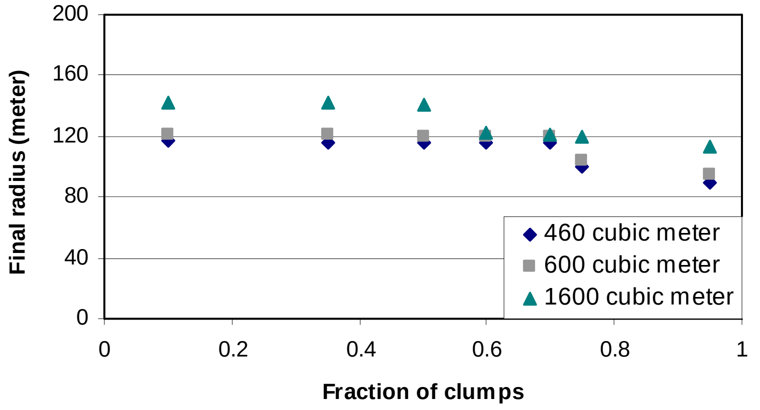

Figure 20 The final radius of the cloud as a function of the amount of clumps. The moisture content is kept constant.

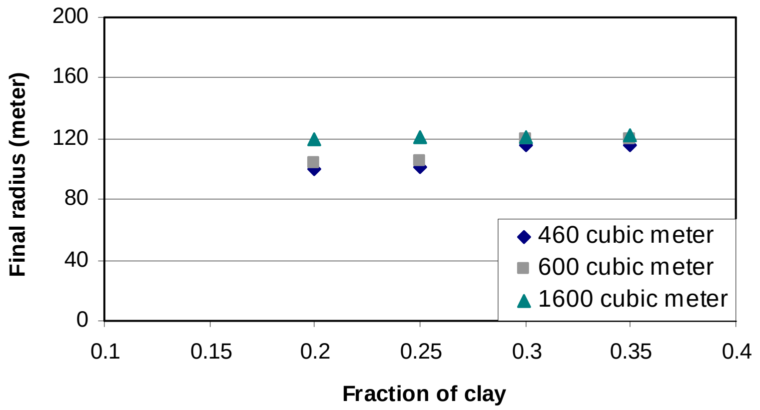

Figure 21 The final radius of the cloud as a function of the amount of clay. The clump fraction is set to a constant value.

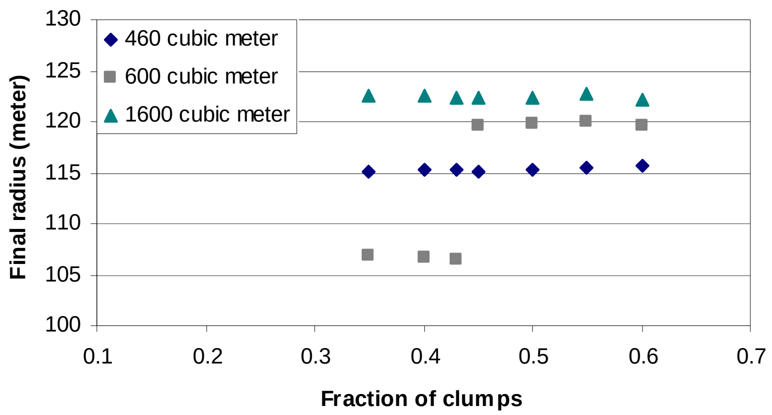

Figure 22 The final radius of the cloud as a function of the amount of clumps. The clay fraction is kept constant.

5.3 SUSPENDED SOLIDS

When the percentage of suspended solids was plotted it was showed that the amount of material that escapes from the cloud increases when disposing material at greater depths (figure 23). The velocity of ambient water does once again not seem to have an effect on the cloud other than convecting it (figure 24).

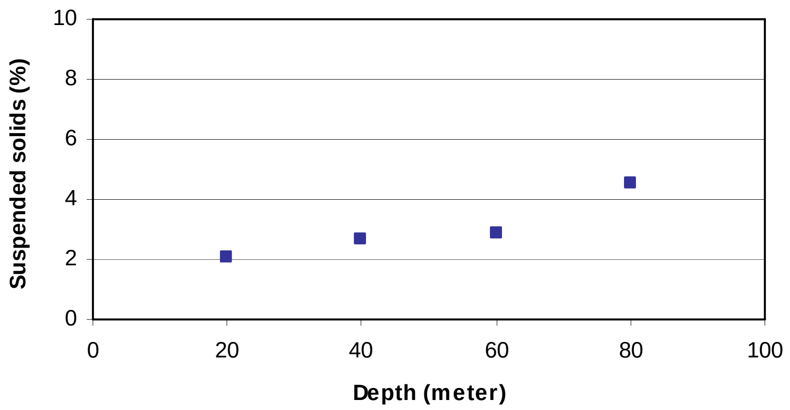

Figure 23 The amount of material suspended in the water column 3600 seconds after disposal as a function of the water depth.

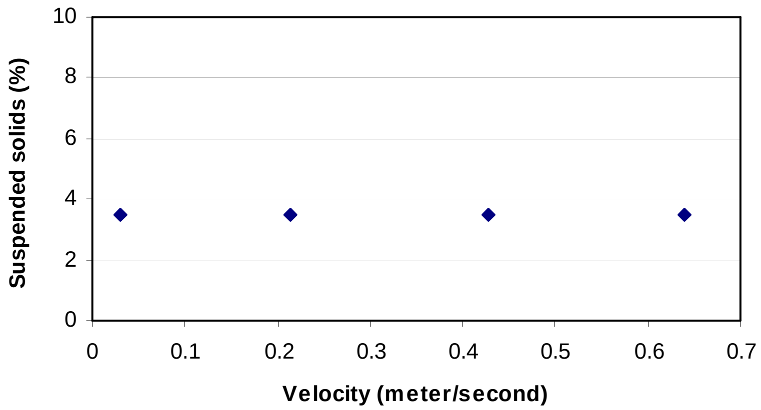

Figure 24 The amount of material suspended in the water column 3600 seconds after disposal as a function of an increased water velocity.

The smallest disposed volume gives a very high amount of solids stripped from the sinking cloud whilst the values for the bigger volumes are quite similar (figure 25).

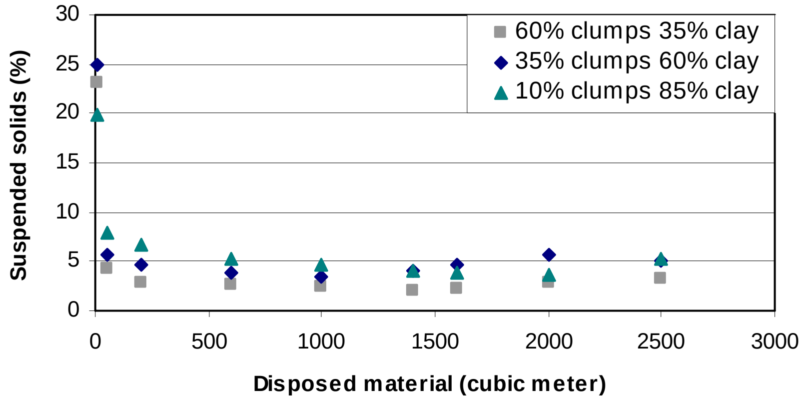

Figure 25 The amount of suspended material as a function of disposed material.

The suspended solid amount goes down with an increased amount of clumps when the moisture content is kept constant (figure 26). When the clay/moisture fraction is plotted in figure 27 it can be seen that as the clay content increases (keeping the clump fraction constant) the suspended solids increase linearly. When the clay fraction is kept constant in figure 28 the suspended solids did not change when the clump fraction was changed.

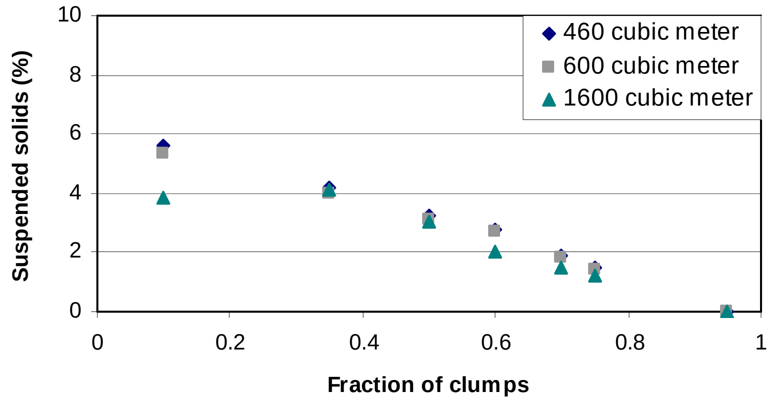

Figure 26 The amount of suspended material as a function of the clump fraction. The moisture content is constant.

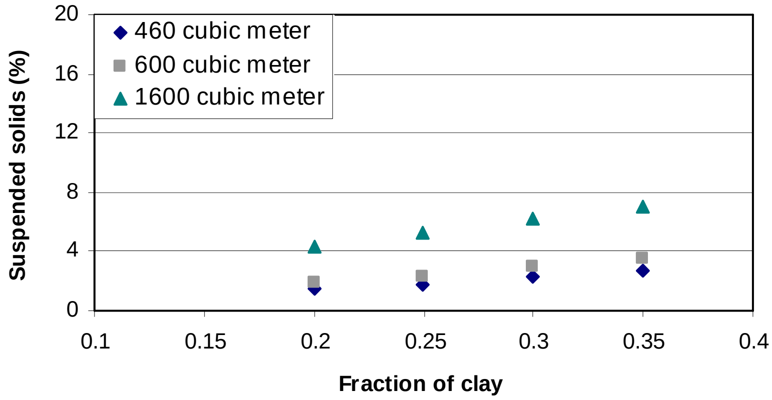

Figure 27 The amount of suspended material as a function of the clay fraction. Since the amount of clumps is constant the moisture content decreases with an increased amount of clay.

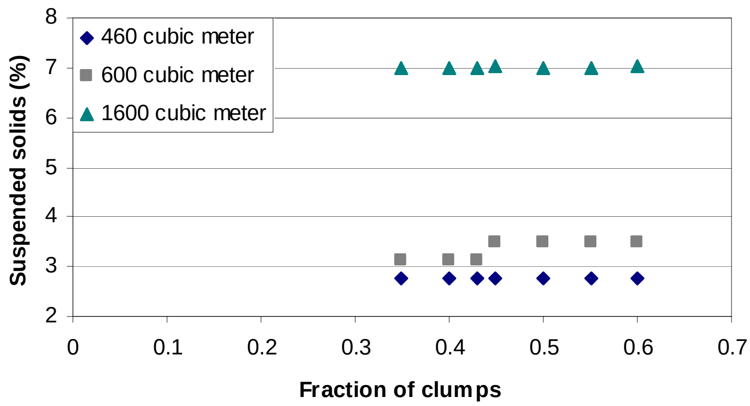

Figure 28 The amount of suspended solids as a function of the clump fraction. The clay content is kept constant.

5.4 MAXIMUM DEPOSITION THICKNESS

The maximum deposition thickness is the thickest part of the material disposed at the bottom, measured after the completion of the collapse phase. The results show that the depth has a small influence on the deposition thickness (figure 29). There is a small decrease in thickness as the water gets deeper. The velocity does not have an effect on the maximum deposition thickness (figure 30). When a larger volume of material is disposed the thickness increases linearly (figure 31).

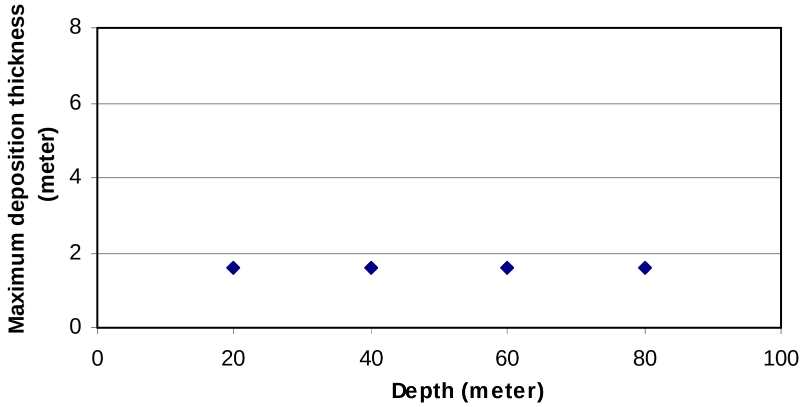

Figure 29 Maximum deposition thickness as a function of depth.

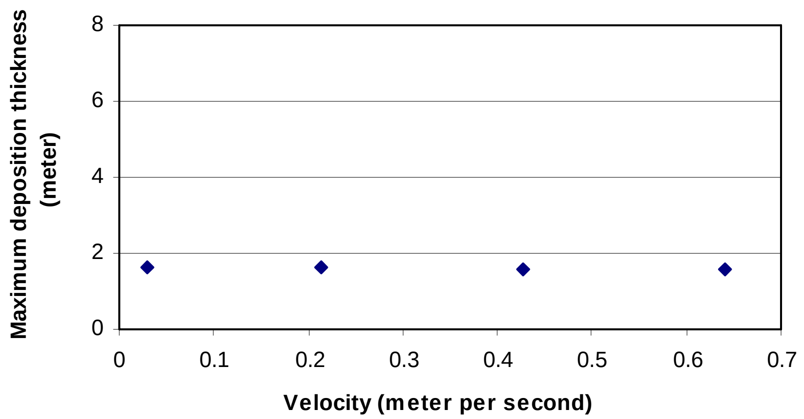

Figure 30 Maximum deposition thickness as a function of the water velocity.

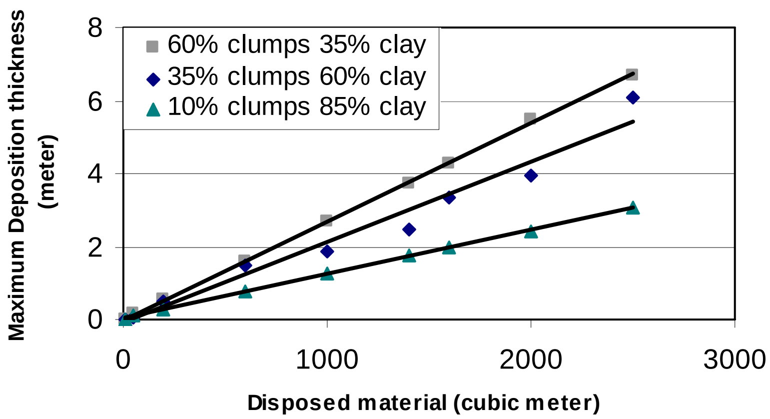

Figure 31 Maximum deposition thickness for different volumes of disposed material and for three different sediment compositions

When the sediment composition was varied and plotted in figure 32 the results show a parabolic shape of the curve. The results from changing the clay/moisture content show a positive linear dependence (figure 33). Figure 34 show that the maximum deposition thickness increases also with an increased amount of clumps and a constant clay fraction.

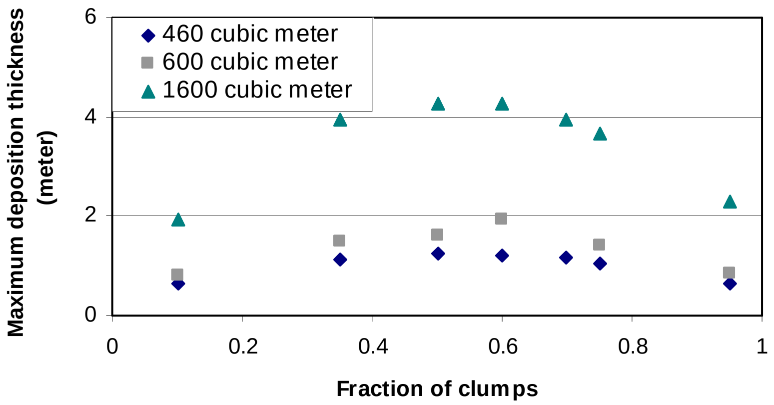

Figure 32 Maximum deposition thickness. The moisture content is constant.

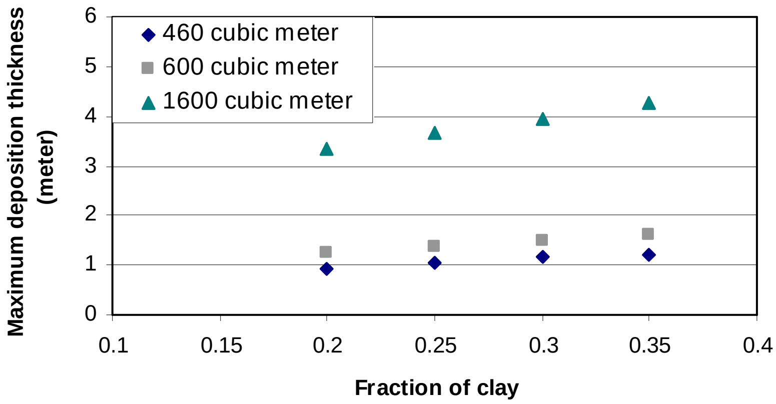

Figure 33 The maximum deposition thickness as a function of the amount of clay. The fraction of clumps is set to a constant value of 0.6.

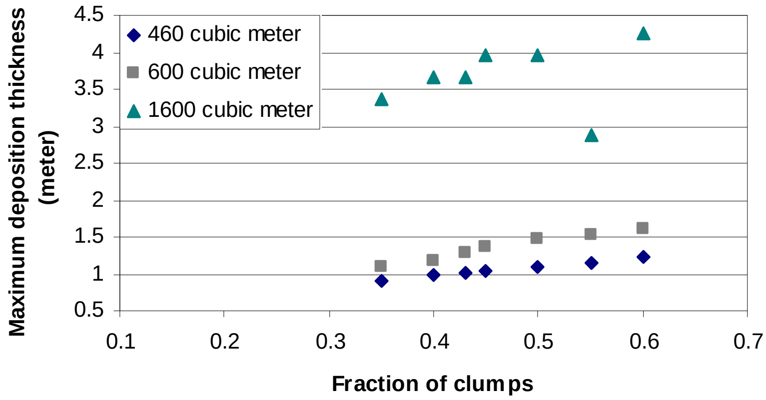

Figure 34 The maximum deposition thickness increases as the amount of clumps increases and the moisture content decreases. The clay content is set to a constant value of 0.35.

6 DISCUSSION

Short-Term Fate has previously been used mostly in the United States. One study was performed by Johnson et al. (1999) at Site 104 in Chesapeake Bay where STFATE was used to estimate the amount of material stripped from the cloud as the dredged material falls through the water column. Another study was performed by Holliday and Johnson (1978) in Duwamish, New York Bight and Lake Ontario. In this case field studies were also performed. They show that correct material characterization in STFATE is very important in obtaining realistic modeling results. It is also shown that the entrainment coefficient is the most sensitive coefficient in the model.

In this project the sensitivities of five different factors in STFATE are tested. By varying one factor at the time keeping the others constant the effect on the sinking material is studied. The distance traveled by the centroid of the material from the barge, the final radius of the material on the bottom, the amount of suspended material after 3600 seconds and the maximum thickness of the material at the bottom were studied and will be discussed in that order in this section.

A complete understanding of the equations describing the model is not within of the scope of this paper. The results will therefore be discussed without reference to them.

6.1 THE DISTANCE TRAVELLED BY THE CENTROID FROM THE BARGE

In conditions where the water is not stratified, where the depth is small and a large amount of material is disposed in one go the material falls straight down very quickly. This makes it hard to draw any conclusions from the results with a varying depth. What can be seen is that the water depth has an influence on the distance and the relation seems to be linear (figure 11). This was expected since with a greater depth the sinking cloud has more time to move horizontally with the current.

The relation between the distance traveled from the disposal site and the horizontal water velocity is also positive linear (figure 12). The distance is much more sensitive to changes in ambient water velocity than to changes in water depth which can be seen by comparing figures 14 and 15. An explanation to this could be that the horizontal water velocity also affects the spreading material after the collapse whilst the depth only has an effect on the convective phase.

There are so small changes in for example figure 14 that the results are difficult to interpret. The smallest volume in figure 13 gives a very long distance. Since the results are based on calculations of complex non-linear equations the results are naturally complex too.

6.2 THE FINAL RADIUS OF THE MATERIAL DISPOSED AT THE BOTTOM

The final radius of the material is defined as the radius at the end of both the convective phase and the collapse phase. In figure 17 it can be seen that the radius grows linearly as the water depth is increased. As the hemispheric cloud descends the water column ambient water is entrained. This makes the cloud grow in volume. The deeper the water is the more ambient water is entrained and the greater the radius becomes.

An increased ambient water velocity does not affect the final radius (figure 18). The cloud is convected with the current without loosing its shape and therefore hits the bottom unchanged.

This is not the case for the simulations where the disposed volumes were varied which can be seen in figure 19. The final radius first grows quite quickly with an increased amount of dredged material disposed. It then seems to reach a plateau where a bigger volume does not seem to affect the final radius that much. Since the radius of the cloud is related to the volume as the volume$^{1/3}$ the radius will not increase much when the volume is increased. This could also be an effect of material piling up instead of spreading evenly on a greater surface.

With an increased ratio of clumps the radius of the material at the bottom decreases (figure 20). First the radius seems unaffected by an increased amount of clumps. Then, for the two smaller volumes, it decreases rapidly. This seems reasonable since the material falls quickly to the bottom.

As for the moisture content figure 21 shows that when increasing the clay fraction at the same time as decreasing the moisture content the final radius gets larger. The reason for this is probably the increased amount of material available.

When varying the clump to moisture fraction the results show little sensitivity to changes (figure 22) for the large volume. This is probably because the clay fraction is kept constant and it seems to be the clay fraction that contributes to the spreading.

6.3 THE AMOUNT OF SUSPENDED MATERIAL AFTER 3600 SECONDS

This parameter is defined as the amount of suspended solids 3600 seconds after disposal. This means that the material has already finished both the convective and the collapse phase and that when interpreting the results both phases need to be considered. It is interesting to study the suspended material after both phases since material is also stripped during the collapse.

When the water depth is increased the amount of suspended solids after 3600 seconds is also increased (figure 23). This is probably because of the longer exposure to ambient water which gives the time for small clouds to be stripped from the main cloud.

The amount of suspended solids does not increase with a higher ambient water velocity (figure 24). The reason for this is that the water velocity only acts to move the sinking material horizontally.

In simulations with increasing disposed volumes (figure 25) the amount of suspended solids for the smallest volume goes up to 20-25% when the other values are at around 5%. This show that disposal of a large volume is better than several smaller to decrease the amount of suspended material. The results were expected since the surface area of the cloud that is exposed to the surroundings is relatively larger for smaller volumes than for larger volumes.

The results from the simulations with a varying ratio of clumps show that the percentage of suspended solids goes down with an increased amount of clumps (figure 26). The high values for low fractions of clumps depend on high amounts of clay since when decreasing the clump fraction the clay fraction automatically goes up.

The sensitivity for changes in the amount of clay is bigger than the sensitivity for changes in the amount of clumps. A verification of this can be made by looking at the particle properties under material description data in STFATE. For all particles it is set if they are stripped during descent or not and for clumps it is set not to. STFATE gives properties to the particle groups like a settling coefficient and a fall velocity.

When the effect of adding more moisture to the sediments was tested at the expense of a decreased amount of clay (keeping the clump fraction constant) there seemed to be a linear dependence (figure 27). When keeping the clay fraction constant and increasing the moisture at the expense of the amount of clumps there was no change (figure 28). This also shows that the amount of clumps is not an important parameter as long as the amount of clay is constant.

6.4 THE AMOUNT OF MATERIAL SUSPENDED AT DIFFERENT DEPTHS

Attempts have been made to plot the amount of material that leaves the sinking plume at different depths during the descent. The stripped material is assumed to behave as clouds of Gaussian distribution (equation 22, appendix 2) that are convected with ambient water currents. The output files give information on for example the time of creation of the cloud, the location, the depth, thickness and total mass.

Results from the output files were manipulated in Microsoft Excel but did not give any clear results. Since the term “total mass” given in cubic feet was not clearly defined it was hard to interpret the results. However one could see that the ambient water velocity did not affect the amount of suspended material and that most of the stripped material, in all cases, seems to go into suspension close to the bottom.

6.5 THE DEPOSITION THICKNESS AT THE BOTTOM

The maximum deposition thickness is defined as the thickest part of the mound created when all material has settled at the bottom. This parameter is interesting since when planning disposal operations the aim is to put the material at a depth where it is not affected by the surroundings. If the material piles up too high it might reach over the edge of the depression where it was put and gets exposed to currents.

The maximum deposition thickness was plotted and the results show that the thickness is not sensitive to the water profile depth (figure 29). The thickness only decreased a very small amount with an increased depth. This is so since the material was spread out on a bigger area.

The results from the runs with varying ambient water velocities show that the velocity does not have an effect on the maximum bottom thickness. Since the final radius and the amount of suspended material do not change with increased water velocities (figures 12 and 24) it is normal that the maximum bottom thickness does not either (figure 30).

It is natural that the deposition thickness would increase with an increasing volume of dredged material being disposed, and the relation seems to be linear (figure 31). Frictional forces at the bottom hinder the spreading material from spreading in an even layer. As the energy of the cloud moving along the bottom loses its energy and stops, material will start to settle on top of the already settled material.

The amount of clumps in the material also affects the maximum deposition thickness. In figure 32 it can be seen that the maximum bottom thickness first increases with an increased amount of clumps but then starts decreasing. It seems like a mixture of clay and clumps would give the thickest deposition on the bottom. The first thought was that so much coarse material can not be stacked but would naturally collapse. The maximum height of the disposed material in reality depends on the steepest angle at which the material can sustain itself from environmental forces and gravity. But since this process is not accounted for in STFATE there might be another explanation to the result.

In the plots where the clay/moisture content was increased the maximum bottom thickness increased too (figure 33). This is because as the clay amount increases the mass of the material increases and more material gives a larger maximum bottom thickness. The simulations with a varying clump to moisture fraction show an increased maximum bottom thickness with increasing amount of clumps (figure 34).

7 CONCLUSIONS AND RECOMMENDATIONS

The aim of this study was to learn which processes might affect the dredged material initially after disposal into open water and how the disposal could be modeled. This was done using STFATE to model a disposal operation varying different parameters while keeping others constant. The results from this project will hopefully be used in future planning of dredged material disposal operations.

The results from simulations show that the ambient water velocity only acts to move the material horizontally. This is still important to take into consideration when choosing a point of disposal since the sinking material could miss the calculated collapse point. The depth is important for the same reasons. A greater depth gives the ambient currents more time to move the cloud out of position.

When using STFATE to model disposals of very small volumes it sometimes gives surprising results. If the smallest volume was disregarded when interpreting the results the disposed volume did not affect the distance traveled by the centroid from the point of disposal, or the amount of suspended solids, but had an effect on the final radius and of the maximum deposition thickness. If the equations are assumed to represent the reality the results show that a higher percentage of solids are stripped from a smaller disposed volume than from a larger volume and that a smaller volume is transported further away from the point of disposal. This should be kept in mind when planning disposal operations.

It was also shown that the model is not sensitive to the amount of clumps but to the amount of clay. This was further shown in the simulations with varying moisture content. When the clump fraction was kept constant and the clay fraction was allowed to vary a linear relationship between the clay fraction and the amount of suspended solids was shown.

If more time would have been available it would have been interesting to vary other sediment properties. It would also have been interesting to see how consolidation during transport affects the spreading of material and use the choice in STFATE of simulating the material as separated into several layers. Other plots could have been created, such as a concentration versus time, to see how fast the suspended material settles.

Other parameters that were kept constant in simulations could have been allowed to vary. For example it would have been valuable with simulations where the ambient water velocities were allowed to vary with depth. One could also have varied the bathymetry.

The collapse phase is still to be looked at and it would have been very interesting to see how much of the material that really goes into suspension in that phase.

STFATE is a program that is easy to use, even without an extensive amount of data and it could be used to get a quick view of possible short term scenarios. One should still bear in mind that it is no more than a model and it gives a quality of the output similar to the quality of the input. The model structure might also influence the results.

It is clear that STFATE can only be used to determine the short term fate of dredged material. STFATE can, with the right input, be used in planning of dredged material disposal operations but with the addition of a long term model or bearing in mind that long term processes such as land lifting, climate changes and future use of the area could change the conditions around the disposed masses. Just as important to make sure that the material settles at the chosen location is that it stays there for a long time afterwards.

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APPENDIX 1 WORD LIST

This is an English to Swedish word list explaining some words that are being used throughout the report.

Ambient = Omgivande

Barge, Hopper, Scow = Pråm eller liknande transportmedel för muddermassor

Buoyancy = Flytkraft

Centroid = Masscentrum

Clamshell = Skopmuddring

Disposal = Tippning

Dredge = Muddra

Entrain = Dra med, dra in

Momentum = Rörelsemängd

Shear forces = Skjuvkrafter

Stratification = Skiktning

Suction = Sugmuddring

The Swedish Board of Fisheries = Fiskeriverket

Turbidity = Grumlighet

APPENDIX 2 DETAILED DESCRIPTION OF THE PROCESSES

This appendix is included for the interested to get a better look at the equations that describe the processes in STFATE. These equations are shortly described but the complete understanding of them is not in the scope of this study.

GOVERNING EQUATIONS

The Convective Phase

The equations governing the convective phase include those for conservation of mass, momentum, buoyancy, solid particles and vorticity. Due to its high density and initial momentum (from the disposal) the hemispheres will sink towards the bottom. Ambient water will be entrained in the plume and some of the really fine particles will be stripped from the sinking plume to ambient water. Equation 1 describes the time rate of change of the mass of the cloud.

$$\frac{d}{dt}(V_c\rho) = E\rho_a - \sum_i S_i\rho_i \qquad (1)$$

The volume of the particle cloud is defined as the volume of a hemisphere (equation 2) where a is the radius of the hemisphere. E is described by equation 3. ρ_a is the density of the ambient water and ρ_i the average particle density of particle fraction number *i*. S_i is the volume rate of flow of particles out from fraction number *i* according to equation 4. (Brandsma, 1976)

$$V_c = \frac{2}{3}\pi a^3 \qquad (2)$$

$$E = 2\pi a^2 \alpha \left|\vec{U} - \vec{U}_a\right| \qquad (3)$$

α the entrainment coefficient, $\vec{U}$ the velocity of the cloud and $\vec{U}_a$ is the speed of the ambient water.

$$S_i = \pi a^2 \left|v_{fi}\right| C_{si}(1-\beta_i) \qquad (4)$$

v_{fi} is the fall velocity, C_{si} is the volume fraction of component *i* in the cloud and β_i is a settling coefficient. Both v_{fi} and β_i are constant for each particle fraction (clumps, sand, silt or clay). Figure 1 shows a sinking dredged material cloud, a velocity profile and a density profile of the ambient water. (Brandsma, 1976)

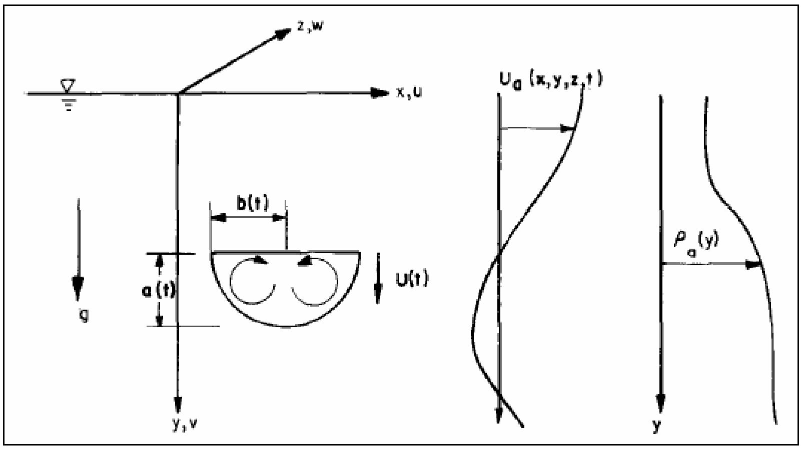

Figure 1 The coordinate system of a sinking cloud in the shape of a hemisphere, a velocity profile and a density profile for ambient water.

The second governing equation is equation 5 which describes the time rate of change of momentum. It is a function of the buoyancy, the drag, the rate of ambient fluid entrainment and the rate of solids leaving the cloud. $\vec{M}$ is described by equation 6 where C_m is an apparent mass coefficient. C_m is recommended to be set to 1.0-1.5. The buoyancy force, F, is described in equation 7. $\vec{j}$ is the unit vector in the vertical direction and the drag, $\vec{D}$, in the three directions x, y, and z is defined in equations 8, 9 and 10. (Brandsma, 1976)

$$\frac{d}{dt}\left(\vec{M}\right) = F\vec{j} - \vec{D} + E\vec{\rho}_a\vec{U}_a - \sum_i S_i\rho_i\vec{U} \qquad (5)$$

$$\vec{M} = C_m\rho\frac{2}{3}\pi a^3\vec{U} \qquad (6)$$

$$F = \frac{2}{3}\pi a^3 g(\rho - \rho_a) \qquad (7)$$

$$D_x = 0{,}5\rho_a C_D(0{,}5\pi a^2)\left|\vec{U} - \vec{U}_a\right|(u - u_a) \qquad (8)$$

$$D_y = 0{,}5\rho_a C_D\pi a^2\left|\vec{U} - \vec{U}_a\right|v \qquad (9)$$

$$D_z = 0{,}5\rho_a C_D(0{,}5\pi a^2)\left|\vec{U} - \vec{U}_a\right|(w - w_a) \qquad (10)$$

C_D is a drag coefficient.

The equation describing the time rate of change of relative buoyancy is shown in equation 11. $\rho_a(0)$ is the density of ambient water at the surface.

$$\frac{dB}{dt} = E(\rho_a(0) - \rho_a) - \sum_i\left(S_i(\rho_a(0) - \rho_i)\right) \qquad (11)$$

Equation 12 is the equation for buoyancy which depends on the volume of the hemispheric cloud and the density difference between the ambient fluid and the cloud.

$$B = \frac{2}{3}\pi a^3(\rho_a(0) - \rho) \qquad (12)$$

The solid volume of component *i* (P_i) changes with time according to equation 13 and depends on the flow of particles out of the cloud.

$$\frac{dP_i}{dt} = -S_i \qquad (13)$$

P_i is described in equation 14 and depends on the volume of the cloud and the volume fraction of that component in the cloud.

$$P_i = \frac{2}{3}\pi a^3 C_{si} \quad (14)$$

The last governing equation is that for vorticity. The total vorticity is the mechanism that helps the cloud to keep its identity. It is also important when determining the amount of entrainment of ambient water. When a cloud of material leaves the sinking plume and passes through the boundary and into the ambient water, it gets an initial vorticity.

The total vorticity is created by shear forces at the boundaries. When the cloud is in the ambient water there are two possibilities, except for bottom contact. In a non stratified environment, with a homogenous density, the cloud keeps its total vorticity although the vortex strength is decreasing with cloud growth. In a stratified fluid the density gradient will act to decrease the total vorticity (equation 15) (Brandsma, 1976). K is vorticity, A is a dissipation parameter defined in equation 16 and ε is the density gradient (equation 17).

$$\frac{dK}{dt} = -A\varepsilon \quad (15)$$

$$A = \frac{Ca^2 g}{\rho_a(0)} \quad (16)$$

$$\varepsilon = \frac{d\rho_a}{dy} \quad (17)$$

C is a vorticity dissipation coefficient that is, according to Turner (1960), set to 3.
Equation 15 is probably more complicated and might have been changed since the Brandsma and Divoky wrote the report in 1976 (Brandsma 1976).

The entrainment coefficient α in equation 3 depends on the properties of the sediment cloud, properties of ambient water and turbulence inside and outside of the cloud.
In studying vortex ring motion Turner in 1960 discovered a coefficient describing entrainment. He did this by assuming similarity where C_l was found to be 0.16. B is buoyancy and K is the total vorticity (equation 18).

$$\alpha = \frac{B}{2\pi\, g\, C_l\, K^2} \quad (18)$$

When the cloud falls and its vorticity approaches zero, the assumption of similarity can not hold. In turbulent thermals it is found that α approaches α0.
Koh and Chang (1973) therefore thought of formulating an expression for α (equation 19) where α was dependent on B and K. α approaches α_0 when vorticity (K) approaches zero. K decreases with increasing depth.

$$\alpha = \alpha_0 \sqrt{\tanh\left(\frac{B}{2\pi g C_l K^2 \alpha_0}\right)^2} \quad (19)$$

Their only justification for this equation is that it tends to the correct limits (to Turner's relation when K is large and to α_0 when K is small.

The dimensionless mass rate of settling is given in equation 20. Koh and Chang (1973) used this expression after doing a dimension analysis that showed that the mass rate of settling is a function of the ratio of the sinking velocity of the cloud (v) and of the solid particles (v_{fi}) and the concentration of each particle group (C_{si}).

$$\frac{q}{v_{fi}\rho_i a^2} = \pi C_{si}(1-\beta_i) \quad (20)$$

β_i is called the settling coefficient and is assumed to depend on v/v_{fi}, C_{si} and C. β_i is expected to vary between 0 and 1 depending on if the particles are settling freely or not settling at all (equation 21). β_0 is assumed to be known. (Brandsma, 1976)

$$\beta_i = \begin{cases} 0 \quad if \quad \left|\frac{v}{v_{fi}}\right| < 1 \\ \beta_0 \quad if \quad \left|\frac{v}{v_{fi}}\right| \geq 1 \end{cases} \tag{21}$$

Given a set of initial conditions these equations can be solved with several numerical methods. The output from these calculations is used as input for calculations on dynamic collapse and long term passive diffusion.

During the convective descent phase fine material may leave the cloud, due to ambient water currents or movement of the barge, and stay in the upper parts of the water column. The material that leaves the sinking plume is considered as clouds with Gaussian particle distribution that are convected by ambient water currents. Experiments have shown that fine particles may also leave the density current at the impact at the bottom. These clouds are also considered to have a Gaussian particle distribution (equation 22). (Johnson, 1990 and Johnson, 1993)

$$C = \frac{m}{(2\pi)^{3/2}\sigma_x\sigma_y\sigma_z}\exp\left\{-\frac{1}{2}\left[\frac{(x-x_0)^2}{\sigma_x^2}+\frac{(y-y_0)^2}{\sigma_y^2}+\frac{(z-z_0)^2}{\sigma_z^2}\right]\right\} \tag{22}$$

The concentration of particles at a chosen position is defined as C in the equation above. *m* is the total mass of the stripped cloud. The standard deviations in the different directions are denoted σ_x, σ_y and σ_z. x, y, z are spatial coordinates and x_0, y_0 and z_0 are coordinates of the cloud centroid.

Dynamic Collapse at the Bottom

The collapse phase was not the focus of this project. For the interested the equations are given below or can be found in the work of Brandsma (1976).

Depending on if the ambient water is stratified or not the cloud will collapse either at the stratification boundary or continue and collapse at the sea bed (Brandsma, 1976). In Sweden dredging operations are mostly performed during autumn when the water is well mixed. For that reason this essay will only discuss the dynamic collapse at the bottom. Further information on collapse at a stratification boundary can be found in the work of Brandsma (1976).

As the sediment cloud is sinking towards the bottom during the convective phase its momentum increases. Simultaneously the particle concentration is decreasing. Its horizontal velocity approaches the velocity of the ambient water and the vorticity approaches zero.
When the cloud collapses at the bottom, its cross section gets elongated in the horizontal direction and is now described as a half ellipsoid (figure 2a) equation 23. After impact the dredged material continues to spread radially until it has lost so much energy that the particles start to settle.

$$\frac{y'^2}{a^2}+\frac{r'^2}{b^2}=1 \tag{23}$$

The sediment cloud is assumed to remain symmetrical and the bottom is assumed to be horizontal. The equations describing collapse at the bottom are practically identical to the ones describing collapse at a stratification boundary except for some changes of geometry and to account for the reaction and friction forces at the bottom. The governing equations are described below (Brandsma 1976).

$$\frac{d}{dt}(V_c\rho) = E\rho_a - \sum_i S_i\rho_i \qquad (24)$$

V_c is defined in equation 25.

$$V_c = \frac{2}{3}\pi ab^2 \qquad (25)$$

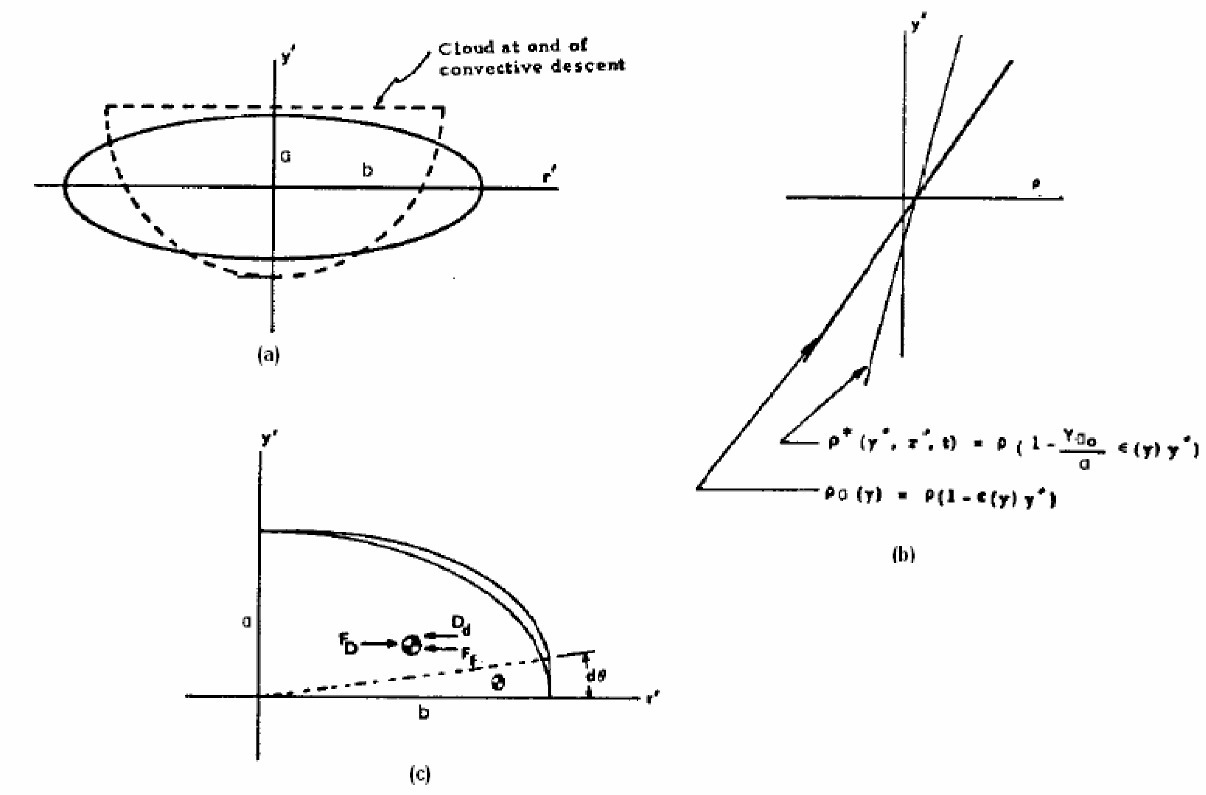

Figure 2 A collapsing sediment cloud. Figure 2a shows how the shape of the cloud changes from a hemisphere to half a spheroid at the bottom contact. 2b shows the density gradients of ambient water (ρ_a) and for the cloud (ρ*) and 2c shows the forces acting on the centroid of the spheroid and a slice of the cloud with its centroid. (Brandsma 1976)

The equation of momentum is identical to equation 5 except for an addition of the reaction force at the bottom (F_f).

$$\frac{d}{dt}(\vec{M}) = F\vec{j} - \vec{D} + E\vec{\rho}_a\vec{U}_a - \sum_i S_i\rho_i\vec{U} - \vec{F}_f \qquad (26)$$

$$\frac{dB}{dt} = E(\rho_a(0) - \rho_a) - \sum_i (S_i(\rho_a(0) - \rho_i)) \qquad (27)$$

$$\frac{dP_i}{dt} = -S_i \qquad (28)$$

The major auxiliary equations describe the entrainment of ambient water, the settling of solid particles and for the collapse of the cloud. The entrainment is due to the collapse alone since the velocity of ambient fluid at the bottom is assumed to be small. The entrainment coefficient is a product of the surface area of the half ellipsoid that is exposed to the ambient water, an entrainment coefficient and of the tip velocity of the collapsing cloud (equation 29).

$$E = \left\{ \pi b^2 + 0{,}5\pi \frac{a^2 b}{R} \ln\left(\frac{b+R}{b-R}\right) \right\} \alpha_c \frac{db}{dt} \tag{29}$$

$$R = \sqrt{b^2 - a^2} \tag{30}$$

α_c is the entrainment coefficient and db/dt is the tip velocity.

$$S_i = \pi a^2 \left| v_{fi} \right| C_{si} (1 - \beta_i) \tag{31}$$

It is density differences between the inside and the outside of the cloud that drives the collapse. The assumption is made that the density gradient on the inside of the cloud is less than that in the ambient fluid by a factor $\gamma a_0/a$, where a is the shorter semi axis and of the half ellipsoid and a_0 is the final radius of the hemisphere (figure 2a). If the assumption is made that the density gradient of ambient water is constant and described by equation 32 (Brandsma 1976)

$$\varepsilon = \frac{1}{\rho} \frac{\partial \rho_a}{\partial y} \tag{32}$$

the density of ambient water (ρ_a) is defined as:

$$\rho_a = \rho_0 (1 - \varepsilon y') = \rho_0 (1 - \varepsilon (a - y)) \tag{33}$$

The density of the cloud is given in equation 34

$$\rho = \rho_0 \left(1 - \gamma \frac{a_0}{a} \varepsilon y' \right) = \rho_0 \left(1 - \gamma \frac{a_0}{a} \varepsilon (a - y) \right) \tag{34}$$

Where y and y' are defined in figure 2b above.

Figure 2c shows the centroid of the collapsing cloud. The centroid of a slice of the collapsing cloud with an angular dimension $d\theta$ is moving with respect to the centroid of the cloud. The forces acting on the centroid are shown. F_D is the internal forces minus the external forces acting on the centroid. D_d is the form drag which is a friction force caused by the shape (form) of the cloud and F_f is the skin friction drag (a friction force depending on the properties of the surface of the cloud). If the thin slice is considered as a free body the external force F_{ext} can be calculated by integrating the pressures over the rounded external surface of the slice (equation 35). F_{ext} is the external radial force, acting at the slice centroid (figure 3). The pressures are assumed to be hydrostatic and the pressure at the cloud boundary is equal to the pressure of the ambient fluid, $p_a(y)$.

$$F_{ext} = \int_{y=0}^{y=a} p_a(y) r(y) d\theta dy \tag{35}$$

The pressure on the inside of the cloud ($p_c(y,r)$) is integrated on both sides of the slice and $d\theta$ is assumed to be small. The internal force (F_{int}) driving the collapse is a function of the radial position inside the cloud and is directed radially outwards (figure 3) (equation 36).

$$F_{int} = \int_0^b \int_{a(1-R)}^{a} p_c(y,r) d\theta dr dy \tag{36}$$

R is defined as:

$$R = \sqrt{1 - \frac{r^2}{b^2}} \tag{37}$$

After subtracting F_{int} - F_{ext} the radial force driving the collapse (F_D) is obtained (equation 38).

$$F_D = \frac{\pi\rho_0(1-\gamma\frac{a_0}{a})\varepsilon g a^3 b}{16} d\theta \quad (38)$$

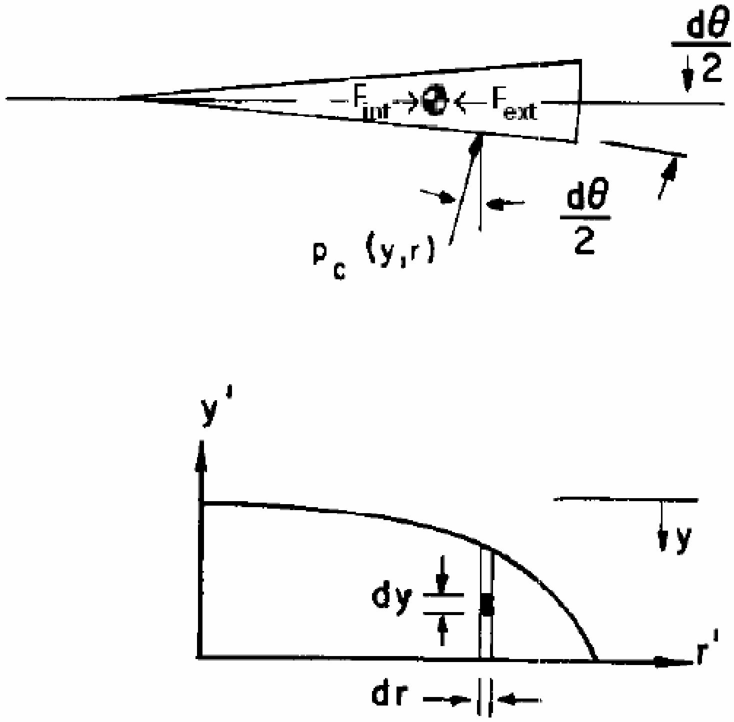

Figure 13 Integration elements for determining the driving of collapse.

The equation describing inertia of the slice is formulated by assuming that the horizontal velocities of the elements inside the slice are related to the radial distance (r) from the centroid of the cloud. It is also assumed that the velocity of horizontal deformation of the cloud is characterized by the velocity of the centroid of the slice. The tip velocity of the cloud (v_1) is linearly related to the velocity of the centroid (dc/dt):

$$v_1 = \frac{16}{3\pi}\frac{dc}{dt} \quad (39)$$

Form Drag (D_D), skin friction drag (F_f) and bottom friction force (F_{bf}) are forces resisting the collapse of the slice.

$$D_D = C_{drag}\rho_a \frac{ab}{4}|v_1|v_1 d\theta \quad (40)$$

$$F_f = C_{fric}\rho_a \frac{b^2}{2a} v_1 d\theta \quad (41)$$

$$F_{bf} = F_b F_{frictn} F_1 \frac{d\theta}{2\pi} \quad (42)$$

C_{drag} is the drag coefficient for a wedge and C_{fric} the friction coefficient for a flat plate. F_{frictn} is the bottom-cloud friction coefficient and F_1 is a modification factor used when calculating the resistance of the friction force of an arc of a half ellipsoid.

The horizontal inertia is the time rate of change of the product between the mass and velocity of the centroid of the slice with a thickness of $d\theta$:

$$I = \frac{d}{dt}\left(\rho \frac{\pi a b^2}{16} v_1\right) d\theta \quad (43)$$

All forces acting on the slice:

$$I = F_D - D_D - F_F - F_{bf} \quad (44)$$

The velocity of the tip of the cloud is the sum of the velocity due to the collapse (v_1) and the velocity due to entrainment of ambient water (v_2) (equation 45).

$$\frac{db}{dt} = v_1 + v_2 \quad (45)$$

The velocity of entrainment is given by:

$$v_2 = \frac{E\rho_a - \sum S_i \rho_i}{\rho \frac{4}{3}\pi a b} \quad (46)$$

Some extra equations are needed to describe the momentum, buoyancy force, and drag forces in the three directions.

$$\vec{M} = C_m \rho \frac{2}{3} \pi a^2 \vec{U} \quad (47)$$

$$F = \frac{2}{3}\pi a^2 g(\rho - \rho_a) \quad (48)$$

$$D_x = \frac{1}{4}\rho_a C_{D_3} (\pi a b)\left|\vec{U} - \vec{U}_a\right|(u - u_a) \quad (49)$$

$$D_y = 0 \quad (50)$$

$$D_z = \frac{1}{4}\rho_a C_{D_3} (\pi a b)\left|\vec{U} - \vec{U}_a\right|(w - w_a) \quad (51)$$

Equation 52 describes the resultant velocity.

$$\psi = \sqrt{u^2 + w^2} \quad (52)$$

The bottom friction force in the X-direction, the bottom reaction force in the y-direction and the bottom friction force in the z-direction are given in equations 53, 54 and 55.

$$F_{F_X} = F_b F_{rictn} \frac{u}{\psi} \quad (53)$$

$$F_{F_y} = -F_b = -\frac{2}{3}\pi a b^2 g(\rho - \rho_a) + \frac{d}{dt}\left(C_M \rho \frac{2}{3}\pi a b^2 v\right) + \pi b^2 \sum_i \left|v_{fi}\right| \rho_i C_{si}(1 - \beta_i) v \quad (54)$$

$$F_{F_Z} = F_b F_{rictn} w / \psi \quad (55)$$

Buoyancy, B, is defined in equation 56 and the solid volume of the i^{th} particle is described in equation 57.

$$B = \frac{2}{3}\pi a b^2 (\rho_a(0) - \rho) \quad (56)$$

$$P_i = \frac{2}{3}\pi a b^2 C_{si} \quad (57)$$

An additional condition is necessary to solve the above equations. The distance between the centroid of the cloud and the base of the half ellipsoid has to be 3/8 of its vertical altitude. (Brandsma, 1976)